UNIVERSITY OF ALBERTA

# An Exploratory Study into the Development of Cognitive and Metacognitive Processes in Mathematics Problem-Solving via Computer

by

DOEHEE AHN

A Thesis Submitted to the Faculty of Graduate Studies and Research in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY

Department of Educational Psychology

Edmonton, Alberta

Fall, 1998

National Library of Canada
Acquisitions and Bibliographic Services
395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque nationale du Canada
Acquisitions et services bibliographiques
395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-34720-6

Canada

UNIVERSITY OF ALBERTA

LIBRARY RELEASE FORM

NAME OF AUTHOR: DOEHEE AHN

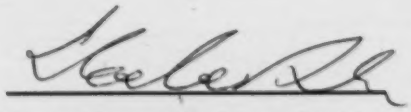
TITLE OF THESIS: An Exploratory Study into the Development of Cognitive and Metacognitive Processes in Mathematics Problem-Solving via Computer

DEGREE: Doctor of Philosophy

YEAR THIS DEGREE GRANTED: 1998

Permission is hereby granted to the University of Alberta Library to reproduce single copies of this thesis and to lend or sell such copies for private, scholarly, or scientific research purposes only.

The author reserves all the other publication and other rights in association with the copyright in the thesis, and except as hereinbefore provided, neither the thesis nor any substantial portion thereof may be printed or otherwise reproduced in any material form whatever without the author's prior written permission.



Lotte GwanAk Tower #1602
ShinDaeBang-2 Dong, 395-67,
26Tong, DongJak-Ku,
Seoul, Korea

Date: Oct 2 1998

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled An Exploratory Study into the Development of Cognitive and Metacognitive Processes in Mathematics Problem-Solving via Computer by Doehee Ahn in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Dr. R. F. Mulcahy (supervisor)

Dr. E. W. Romaniuk (supervisory committee)

Dr. R. H. Short (supervisory committee)

Dr. C. R. Yewchuk (examining committee)

Dr. T. E. Kieren (examining committee)

Dr. W. J. Hunter (external examiner)

Date: Sept 24 /98

## Abstract

In order to enhance students' mathematics problem-solving skills, many researchers have asserted that strategies should be taught more explicitly and specifically; however, there is still a relative lack of research on the combination of general and specific strategies to enhance cognitive and metacognitive processes in mathematics problem-solving skills.

This study, utilizing a general strategy, SPIDER, and two types of computer-based instruction (CBI) programs developed by the researcher, examined the effects of explicit strategies instruction on 105 sixth-grade students' mathematics problem-solving skills and on their attitudes toward mathematics problem-solving.

Students drawn from five public elementary schools were divided into three groups (experimental, CBI control, and traditional control) and further sub-divided on the basis of their mathematics problem-solving achievement levels (high and low).

The students were given the Mathematics Problem-Solving Process Questionnaire (MPSPQ) to measure their perceptions of mathematics problem-solving processes. They were also given the Mathematics Problem-Solving Test (MPST) which was developed for this research as well as the mathematics problem-solving sub-test of the Canadian Test of Basic Skills (CTBS-MPS) to measure their mathematics problem-solving performance. In order to measure their estimation of performance on the MPST, self-prediction of performance (SPP), self-evaluation of performance (SEP),

accuracy of self-prediction of performance (ASPP), and accuracy of self-evaluation of performance (ASEP) were measured. An attitude survey was administered to investigate their attitudes toward mathematics, mathematics problem-solving, computer use, and the CBI program which they received.

The findings indicated that students in the experimental group (n=39) who were exposed to explicit strategies instruction via computer showed a significantly greater improvement on the CTBS than those in the CBI control (n=29) or traditional control (n=27) groups. However, there were no significant differences observed on the MPST, SPP, SEP, ASPP, ASEP, and MPSPQ across the three groups. With respect to the attitude measure, it was observed that students who were exposed to the CBI program either with or without explicit strategies instruction developed more positive attitudes toward mathematics problem-solving. Interestingly, some students in the experimental group showed increased self-confidence in mathematics problem-solving and self-esteem, whereas such comments were not observed from students in the CBI control group.

**To my parents**

**for their love**

**and unconditional support**

**throughout my life**

## Acknowledgment

I would like to express my appreciation to the following individuals who assisted me in bringing this dissertation to its conclusion:

To my supervisor, Dr. R. F. Mulcahy, for his guidance and continual support.

To my supervisory committee members, Dr. R. H. Short and Dr. E. W. Romaniuk, for their encouragement and invaluable comments.

To my external examiner, Dr. W. J. Hunter, for his time and comments.

To my examining committee members, Dr. T. E. Kieren, for his interest and suggestions, and Dr. C. R. Yewchuk, for her encouragement and enormous support as well as insightful comments.

To Dr. J. R. McGregor for his assistance in developing a measure of consistency for this study.

To all the students and teachers who were involved in this study for their cooperation and enthusiasm.

To Ms. A. Wood for reviewing the CBI program prior to its implementation.

To Mrs. W. Lacroix for assisting in the data collection for this study.

To my friends, especially, Dr. O. Stiramon and Ms. S. J. Chen for their encouragement, friendship, and invaluable support.

To my sister and brother for their unconditional support throughout the years.

## TABLE OF CONTENTS


**I Introduction** ..... 1

Definitions ..... 5

**II. Literature Review** ..... 7

Metacognition ..... 7

Metacognitive Knowledge ..... 9

Metacognitive Experience (Self-Regulation) ..... 13

Cognitive and Metacognitive Processes in Mathematical Problem-Solving ..... 18

Cognitive Processes ..... 18

Metacognitive Processes ..... 21

Strategies Instruction in Mathematical Problem-Solving ..... 24

Using Computers to Foster Strategies Instruction ..... 31

**III. Rationale and Research Questions** ..... 39

Rationale ..... 39

Research Questions ..... 42

**IV. Method** ..... 44

Research Design ..... 44

Selection of Subjects ..... 46

Instructional Design and Development ..... 48

Problem-Solving Strategies Embedded CBI Program (PSS-CBI) ..... 54

Screen design....54
Lesson contents....61
Introduction....62
SPIDER....62
Specific strategy....64
Practice....64
Quiz....71
Result....71
Control-CBI Program (Control-CBI)....71
Instrumentation....72
Canadian Test of Basic Skills -- Mathematics Problem-Solving (CTBS-MPS)....72
Mathematics Problem-Solving Test (MPST)....73
Metacognitive Measures....75
Mathematics Problem-Solving Process Questionnaire (MPSPQ)....79
Attitude Survey....80
Procedure....81
Data Analysis....82
**V. Results**....85
Comparison of Student Performance by Mathematics Problem-Solving Achievement Group (High and Low)....86

Reliability Measures on Scoring Procedures of Pre- and Post-MPSTs....87
Performance by Group by Achievement Group....89
Research Question 1....89
Research Question 2....95
Research Question 3....96
Research Question 4....98
Research Question 5....99
Research Question 6....100
Summary....101
Attitudes....102
Research Question 7....102
Research Question 8....104
Comments on the CBI program....107
Summary....112
**VI. Discussion**....114
Observations To Be Considered....122
**VII. Conclusion**....124
Summary of Findings....124
Recommendations for Future Research....127
Limitations and Delimitations....129
**References**....131

**Appendix A**.....146

**Appendix B**.....154

**Appendix C**.....162

**Appendix D**.....178

## LIST OF TABLES


Table 1 Sample Size for Each Group....48

Table 2 Project Timeline....50

Table 3 Scoring Scales for Pre- and Post- MPSTs....74

Table 4 Consistency Matrix for $4(i-1)+j>10$....78

Table 5 Consistency Matrix for $4(i-1)+j\leq 10$....78

Table 6 Conversion of the Consistency Matrix into Decimals....79

Table 7 Research Design....84

Table 8 Performance Differences of Six Pre-test Measures....87

Table 9 Means and Standard Deviations of the Pre- and Post- MPST Scores....88

Table 10 A Summary of Means and Standard Deviations for the Pre- and Post-tests on Dependent Variables....90

Table 11 Means and Standard Deviations of the Pre- and Post- CTBS-MPS Scores....92

Table 12 Means and Standard Deviations of the Pre- and Post- MPST Scores....94

Table 13 Means and Standard Deviations of the Pre- and Post- Self-Prediction of Performance (SPP) Scores....95

Table 14 Means and Standard Deviations of the Pre- and Post- Self-Evaluation of Performance (SEP) Scores....97

Table 15 Means and Standard Deviations of the Pre- and Post- Accuracy of Self-Prediction of Performance (ASPP) Scores....98

Table 16 Means and Standard Deviations of the Pre- and Post- Accuracy of Self-Evaluation of Performance (ASEP) Scores....99

Table 17 Means and Standard Deviations of the Pre- and Post- MPSPQ Scores....100

Table 18 A Summary of Chi-Square Analyses for the Three Groups on Attitudes toward Mathematics, Mathematics Problem-Solving, and Computer Use....103

Table 19 A Summary of Chi-Square Analysis for the Experimental and CBI Control Groups on Attitudes toward Mathematics Problem-Solving and the CBI Program....105

Table A1 Analysis of Variance for the CTBS-MPS with Repeated Measures on Time Factor....147

Table A2 Analysis of Variance for the MPST with Repeated Measures on Time Factor....148

Table A3 Analysis of Variance for Self-Prediction of Performance (SPP) with Repeated Measures on Time Factor....149

Table A4 Analysis of Variance for Self-Evaluation of Performance (SEP) with Repeated Measures on Time Factor....150

Table A5 Analysis of Variance for Accuracy of Self-Prediction of Performance (ASPP) with Repeated Measures on Time Factor....151

Table A6 Analysis of Variance for Accuracy of Self-Evaluation of Performance (ASEP) with Repeated Measures on Time Factor.....152

Table A7 Analysis of Variance for the MPSPQ with Repeated Measures on Time Factor.....153

## LIST OF FIGURES


Figure 1 Interaction between the cognitive and metacognitive processes....29

Figure 2 Recursive and reflective activities of mathematical problem-solving....29

Figure 3 Flowchart for Problem-Solving Strategy (PSS)-CBI program....52

Figure 4 Flowchart for CBI-Control program....53

Figure 5 Main title screen for CBI program....55

Figure 6 Specific title screen for learning strategies component....56

Figure 7 Login screen....57

Figure 8 Screen design....58

Figure 9 Screen explaining first step of SPIDER....65

Figure 10 Screen explaining second step of SPIDER....66

Figure 11 Screen explaining third step of SPIDER....67

Figure 12 Screen explaining fourth step of SPIDER....68

Figure 13 Screen explaining fifth step of SPIDER....69

Figure 14 Screen explaining sixth step of SPIDER....70

Figure 15 Pre- and Post- CTBS-MPSs for Experimental, CBI Control, and Traditional Control Groups....92

Figure 16 Pre- and Post- CTBS-MPSs for High and Low Achievement Groups....93

Figure 17 Pre- and Post- MPST Performance for High and Low Achievement Groups....94

Figure 18 Pre- and Post- SPP Results for High and Low Achievement Groups....96

Figure 19 Pre- and Post- SEP Results for High and Low Achievement Groups....97

Figure 20 Pre- and Post- MPSPQ Results for High and Low Achievement Groups....101

# I. Introduction

Teaching and learning mathematics problem-solving has become a major concern of mathematics educators over the last two decades. It has been associated with various cognitive education programs aimed at improving students' cognitive and metacognitive skills so that they may become more independent learners and better problem solvers. Whether students can become more independent and effective problem solvers is a question central to educational research on cognitive and metacognitive strategy instruction (Charles & Lester, 1984; Derry & Kellis, 1986; Dominowski, 1992; Goldman, 1989; Montague, 1988, 1991, 1992; Montague & Bos, 1986; Okamoto & Kitao, 1992).

Educational programs should provide the context in which concepts and skills such as problem-solving can be learned. Students should acquire various ways to represent problems and should possess a number of strategies to arrive at their solution. In addition, students should learn to value the process of solving problems as much as they value the solutions (National Council of Teachers of Mathematics [NCTM], 1989, p. 23). When teachers take time to explain how problem-solving strategies work and why they are beneficial, students may acquire metacognitive knowledge about memory and the learning process itself (Derry, 1990).

Teaching and learning cognitive and metacognitive strategies is not easy for a number of reasons. First, there are numerous factors that may influence the learner's progress and the teacher's efforts (Hatfield, 1984). Individualized teaching strategies may effectively enhance the learner's performance. However, the teaching of strategies requires a large amount of effort by the teacher, particularly to accommodate students at different ability levels. Second, the teaching of strategies depends heavily on a teacher's ability to provide a structured presentation of explanations and examples, as well as his or her enthusiasm to develop new strategies and integrate them within the regular school curriculum (Hofmeister, 1990). Without teachers' effort and enthusiasm, it is difficult to develop explicit examples of how to teach children to solve problems, how to scan their existing schema for a range of viable strategies, how to match task demands to the strategy set in order to select the best strategies and how to verify and revise the initial strategy selection (Borkowski, 1992).

In the International Commission on Mathematical Instruction *(ICMI)* study (1986), Howson and his colleagues noted the following teaching conditions which prevent teachers from implementing well organized instruction:

> A teacher who has to teach 5 or 6 classes a day has little time to reflect on his or her teaching, to prepare instructional material, to inspect a variety of alternative approaches, or to work with colleagues in developing and renewing a coherent program. A teacher who lacks a

> work space, equipment, a budget for purchasing materials or traveling, discretionary time for working with colleagues or students, cannot be expected to develop far as a 'reflective practitioner' (p. 80).

The question that arises, then, is how can the teaching of cognitive and metacognitive skills be effectively applied to a classroom setting in the face of all the factors described above? One possible way is to use a computer to teach these skills because the teacher can no longer rely solely on a chalkboard, chalk, paper, pencils, and textbooks (NCTM, 1989). Computers can provide greater degrees of flexibility to activate students' learning processes. For example, computers can provide well-structured, individualized, and user-friendly instructional presentations that can facilitate learning. Silver (1985) suggested that the "computer can be used as a tool to create environments in which students can be given the opportunity to think mathematically and solve challenging problems,...can be beneficial in the teaching and learning of problem-solving,...can be designed to provide useful feedback or help sequences that focus not simply on the calculation of an answer but also on processes or heuristics generally useful in problem-solving" (p. 263). In addition, motivation will be enhanced since students can learn at their own pace without group competition and evaluation. However, teaching mathematics problem-solving in a computer-based learning environment has not been done in many research studies.

In this research, the following literature review will investigate the processes of cognition and metacognition involved in mathematics problem-solving. It focuses on the potential for applying computer-based instruction (CBI) in exploring cognitive and metacognitive processes in mathematics word problem-solving as well as the instructional design best suited for improving such problem-solving.

## Definitions

For the purpose of this research, the following definitions have been developed:

1. Problem-solving refers to the process by which an individual uses previously acquired knowledge and strategies to meet the demands of an unfamiliar situation.
2. Metacognition refers to the activation of self-monitoring during mathematics problem-solving performance; that is, self-prediction and self-evaluation of performance made before and after problem-solving performance.
3. Self-prediction of performance refers to the students' prediction of whether they will produce the correct solution or incorrect solution before they try to solve the problem. Accuracy of self-prediction of performance in this study reflects knowledge of cognition (Slife, Weiss, & Bell, 1985).
4. Self-evaluation of performance refers to students' confidence about their solution after trying to solve the mathematics word problem. Accuracy of self-evaluation of performance in this study reflects the regulation of cognition (Slife et al., 1985).
5. General strategy refers to a global term describing a strategy that involves breaking the problem into subproblems which is useful in searching for a problem solution. SPIDER, which was derived and

modified from Polya's (1957) four step model, was used as a general strategy.

6. Specific strategies refer to tools for solving a particular problem. Make a Table, Work Backwards, and Use Logical Reasoning are examples of specific strategies.

# II. Literature Review

This chapter is divided into four parts. In the first part, the definition of metacognition is clarified in mathematics contexts and also research on teaching mathematics problem-solving is reviewed. In the second part, the underlying metacognitive and cognitive processes for mathematics problem-solving are examined. In the third part, previous research on the effectiveness of strategies instruction in mathematics problem-solving is reviewed. In the final part, the rationale for using computers to teach mathematics problem-solving strategies is discussed.

## Metacognition

Kilpatrick (1985) suggested that teachers need to help students become more conscious of what they are doing when they learn and recognize when a learning-related procedure is appropriate. The process is referred to as metacognition. Metacognition influences encoding and retrieval of information in mathematical problem-solving (Silver, 1982). As problem-solving progresses, the learner may make inferences about the meaning of the problem; especially how it can be represented conceptually or graphically. The learner may choose to employ various cognitive strategies to assist in finding the solution. These decisions are essentially metacognitive in nature and influence the way in which information from the problem is encoded

and what other information is retrieved from memory (Jones, Palincsar, Ogle, & Carr, 1987; Schoenfeld, 1985; Silver, 1982).

There is considerable confusion about what metacognition is. Brown (1987) commented:

> The term metacognition came into vogue sometime around 1975. The term has been problematic from its inception, denounced as fuzzy and faddish, and even unnecessary (Marshall & Morton, 1978). It is true that metacognitive-like concepts are fraught with some of the most difficult and enduring epistemological problems of psychology.... it is often difficult to distinguish between what is meta and what is cognitive.... (p. 66).

For clarification, Flavell's (1976) concept of metacognition is as follows: "'Metacognition' refers to one's knowledge concerning one's own cognitive processes or anything related to them, e.g., the learning-relevant properties of information or data... Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective" (p. 232). In the 1970s, Flavell and his colleagues conceptualized children's metamemory. It was conceptualized as what children know about memory, and how they come to know it. As a result of his studies, Flavell has become known as the founder of the term

"metacognition." Metacognition was initially defined as "knowledge and cognition about cognitive objects," that is, about anything cognitive (Flavell, 1987). It also referred to the self-monitoring, regulation, and evaluation of cognitive activity. As well, he asserted that children reflect on their own memory processes. In this section, some aspects of metacognition -- metacognitive knowledge and metacognitive experience -- are reviewed in relation to mathematics problem-solving.

## Metacognitive Knowledge

Metacognitive knowledge is divided into three categories: person, task, and strategy knowledge. Flavell (1987) suggested that these three categories are variables which could influence a person's performance. Garofalo (1985, 1986) pointed out that these variables could influence one's performance in problem-solving.

Person variables refer to one's knowledge about what individuals are like as cognitive (such as affective, motivational, and perceptual) organisms. There are three categories to explain person variables: intraindividual, interindividual, and universal. An intraindividual person variable refers to the belief individuals have about themselves. For example, knowing that you are better at mathematics than at reading is an example of intraindividual person metacognitive knowledge. On the other hand, interindividual variables are comparisons between, rather than within, individuals. Some examples are knowing you are better at mathematics than

your friends, or that your teacher is more thoughtful than the principal. Finally, universal person variables refer to the acquired knowledge we gain from living within a culture. Examples of these variables are your knowledge that you are aware that sometimes people make a mistake but sometimes they do not, or that what you can not recall now you may be able to recall later and vice versa. More generally, you have learned that the human mind is a somewhat unpredictable and unreliable cognitive device (Flavell, 1985, p. 105). Flavell believed that human beings acquire metacognitive knowledge about people that aids the management of their lives. Identifying one's weaknesses and strengths in mathematics, realizing that one makes a lot of careless mistakes, and recognizing that practising always helps in learning algorithms better are all examples of person variables (Garofalo, 1985, 1986).

Garofalo (1985, 1986) and Garofalo and Lester (1986) suggested that in mathematics, person variables should include beliefs about one's own mathematical ability, affective characteristics (e.g., motivation and anxiety), and the relationship between mathematics achievement and achievement in other subject areas. Research has suggested that components of a person's belief system are derived from cognitive experience (Borkowski, 1992; Borkowski, Estrada, Milstead, & Hale, 1989; Silver, 1982). Moreover, a person's beliefs about schooling, learning, and problem-solving in general, and beliefs about mathematics and mathematics problem-solving in particular, can act as powerful guides in encoding and retrieving mathematical material (Silver, 1982) because they influence the decisions

made during problem-solving (Lester, Garofalo, & Kroll, 1989). Borkowski et al. (1989) also suggested that a person's positive beliefs about the importance of effort enhance one's problem-solving skills.

Task variables include one's beliefs about mathematics and the nature of mathematical tasks; this includes knowledge of the scope and demands of tasks and an awareness of the effects of task features (e.g., content and structure) on difficulty (Garofalo, 1985). This is reflected, for example, in the comments: "I have a harder time solving word problems than I do solving computational exercises," and "Some mathematics problems have more than one correct answer." Flavell (1987) argued that the nature of the task dictates certain cognitive rules. If the information is difficult, dense, and without much repetition, the high metacognitive learner knows "to proceed more carefully, slowly, and to process deeply and self-critically" (p. 22).

Strategy variables include knowledge of algorithms[1] and heuristics[2], but also include an awareness of strategies useful in comprehending problems, organizing information, planning solutions, monitoring progress, evaluating results, and knowing when and where to apply them; that is, knowing how and when they should be invoked to facilitate performance (Garofalo, 1985, 1986; Garofalo & Lester, 1985). Examples of this knowledge

---

[1]Algorithms refer to specific step-by-step procedures that guarantee solution when correctly applied. Examples are procedures for performing arithmetic operations such as multiplication and division on multidigit numbers (see English & Halford, 1995; Garofalo, 1986).

[2] Heuristics refer to more general problem-solving procedures that facilitate problem-solving but do not guarantee solution. Examples include "hints" such as controlling the center of the board in a game of chess, looking for analogies, establishing subgoals, examining special cases, and working backwards in mathematical problem-solving (see English & Halford, 1995; Garofalo, 1986).

are the comments: "I know that guessing and checking can help me solve problems that present large numbers or any pieces of data or that asks the solver to find one solution but not possible solutions to a problem," "I know that when I'm stuck on a problem it sometimes helps me to first solve a simpler version of the problem," and "I know that taking my time, working slowly, and checking my steps will lead to more correct answers." (Garofalo, 1986).

Flavell (1987) distinguished between cognitive and metacognitive strategies. He provided the following definitions of cognitive and metacognitive strategies: "A cognitive strategy is one designed simply to get the individual to some cognitive goal or subgoal, such as adding numbers; whereas the metacognitive strategy is for monitoring cognitive progress, such as reading the numbers to check the answer." (p. 23). The term cognitive monitoring has been used synonymously with metacognitive strategy variables. Some young children defined cognitive monitoring as 'spying on your own thinking', and 'getting in touch with your own thinking' (Manning, 1991).

In reality, these three variables (e.g., person, task, and strategy) are not clearly categorized because they interact with each other when an individual assesses one's own progress on a particular mathematical task. For example, Jane knows that she is not good at complicated word problems (task variable), and decides to recheck her solution to make sure if it is correct (strategy variable) because she is not as good in mathematics as in reading (person

variable). Metacognitive knowledge seen above includes information about individuals as learners, the kinds of tasks they encounter, and the strategies they employ to influence the outcome of cognitive activities. To some extent, all learners possess knowledge within these categories which is learned either directly through instruction or from experiences over time.

Metacognitive Experience (Self-Regulation)

In addition to metacognitive knowledge, Flavell (1985) also refers to metacognitive experience. Metacognitive experiences, which may occur at any time before, during, or after a cognitive endeavor, are momentary thoughts, feelings, or realizations which occur while one is working through a cognitive task. Some examples of metacognitive experience statements are: "I don't know what the problem is" and "This method is not helping me solve the problem" (Garofalo, 1986). Metacognitive experience is likely to occur when people are very careful and highly cautious in cognitive tasks (Flavell, 1979). Brown (1987) identified metacognitive experience as the self-regulation required to successfully complete a task. Self-regulation refers to a variety of activities, such as checking, planning, evaluating, testing, and revising. Collectively, these activities are referred to as cognitive monitoring (Billingsly & Wildman, 1990). Since metacognition has to do with awareness, its development requires one to observe what one does and to reflect on what one observes. Thus, students must become watchers, analyzers, assessors, and evaluators of their own mathematical knowledge and behaviors (Callahan &

Garofalo, 1987). Many studies have reported that children do not routinely analyze information provided in the problem nor do they monitor progress and validate the solutions (Garofalo & Lester, 1985; McCoy, 1994; Slife et al., 1985). In Hart's (1993) study with 12 seventh-grade students, the majority appeared to show a lack of individual monitoring or regulation of cognitive activity which is one of the factors impeding problem-solving performance. This relative lack of metacognitive behaviors was also observed even at the college level. Schoenfeld (1985) noted that there was an absence of "periodic monitoring and assessment of solutions" among college students, subsequently resulting in their failure in mathematics problem-solving.

With respect to self-monitoring, some research has examined the accuracy of self-estimation of performance (Pressley, Levin, Ghatala, & Ahmad, 1987; Pressley & Ghatala, 1989; Powel, Morelli, & Nusbaum, 1994; Powel & Gray, 1995). Pressley et al. (1987) and Pressley and Ghatala (1989) found that older grade-school children are more accurate than younger grade-school children in monitoring their performance during test taking. They also argued that younger grade-school children's less accurate estimation is due to less experience with formal school-type testing because accuracy in prediction is an experience based skill. These findings were also shown in Powel's studies (Powel, Morelli, & Nusbaum, 1994; Powel & Gray, 1995) with five-year-old and first-grade children. They found that older children were more accurate in their performance predictions than were younger children.

From these studies on self-prediction of performance, it seems that children's accuracy of prediction improves through experience and maturation.

Some studies have suggested that self-regulated learning can be enhanced by explicitly teaching strategies such as planning and monitoring (Okamoto & Kitao, 1992; Paris & Oka, 1986). More precisely, teaching students directly about what the strategies are, how they work, when they are beneficial and when they are not, and why they are useful to promote learning can help students become more self-regulated learners (Paris & Oka, 1986).

According to cognitive models of learning, learners act upon information to be acquired by relating it to existing knowledge, by imposing structure on incoming information, and by routinely using a variety of encoding and retrieval strategies. Learners exercise much responsibility for their own learning; therefore cognitive models must also include some means for learners to control and regulate their learning. .Self-regulation can be related to "executive skills (i.e., planning, monitoring, controlling, selecting, and evaluating)" which are relevant to mathematical performance, particularly problem-solving, and are also trainable (Sternberg, 1982).

Campione (1987) purported that it is helpful to teach self-regulatory activities in the cognitive domain, such as continually monitoring progress through some task or questioning whether they remained on task.

Garofalo (1986) and Kluwe (1987) argued that self-regulation is influenced by one's metacognitive knowledge even though self-regulation is trainable. For example, a child, believing he or she tends to make

computation mistakes, proceeds cautiously during the computation part of problem-solving and rechecks the answer.

Both of these aspects of metacognition appear to partially overlap each other. That is, self-regulation can affect one's metacognitive knowledge base by observation, comparison, and revision. Garofalo (1986) also noted that metacognitive knowledge is sometimes dependent upon self-regulation and self-regulation is sometimes dependent upon one's metacognitive knowledge.

Furthermore, both of these aspects of metacognition -- metacognitive knowledge and self-regulation -- seem to be crucial in problem-solving and are closely tied to affective concerns. For example, knowledge about cognition is related to expectations regarding success and failure, an important part of student confidence, and the regulation of cognition are related to issues of persistence in problem-solving (McLeod, 1989).

Garofalo and Lester (1985) posited that metacognition seems to be particularly susceptible to influences. For example, decisions made during problem-solving are influenced by an individual's beliefs (Lester et al., 1989). Borkowski, Johnston, and Reid (1986) postulated a bidirectional relationship between attribution and metacognition. They noted that "Good performance following strategy use strengthens general strategic knowledge, which promotes positive self-esteem and attributions of success to effort rather than to uncontrollable factors such as ability or luck. In turn, positive self-esteem

and effort-related attributional beliefs enhance the likelihood of strategy generalization" (Borkowski, Carr, Rellinger, & Pressley, 1990, p. 58).

In contrast to the view of Borkowski and his colleagues on motivational aspects of metacognition, Slife, Weiss, and Bell (1985) believed that motivational factors may be the effects of inadequate metacognitive skills, rather than causes. They noted, in a study which investigated metacognition and cognition in mathematics, that two groups (learning-disabled (LD) and regular groups) "seemed to have comparable motivation in the cognitive task (i.e., performing the same abilities), but they differed on the metacognitive task." That is, LD students could have low motivation or self-esteem because of inadequate metacognitive skills. However, it should be noted that Slife et al. did not examine the motivational aspect in metacognition at least explicitly, whereas Borkowski et al. (1990) examined the motivational aspect specifically in metacognition.

Paris and Winograd (1990) posited the importance of motivational consequences. They mentioned that metacognitive aspects in learning should not be emphasized exclusively, but should equally focus on students' motivational consequences. McLeod (1994) also suggested a more integrated approach to research on affect and cognition in problem-solving. Developing a theoretical perspective which incorporates these ideas is an important task for research on problem-solving (McLeod, 1989).

In order to teach and enhance students' mathematics problem-solving skills, many researchers and educators focus on these aspects of

metacognition. Metacognitive and cognitive processes in mathematical problem-solving are reviewed in the following section.

## Cognitive and Metacognitive Processes in Mathematical Problem-Solving

Many psychologists assert that problem-solving is the highest level of thinking or learning skills. Crucial to problem-solving ability are the learner's cognitive and metacognitive skills.

There has been extensive research on cognition and metacognition in mathematics problem-solving with learning disabled students (Borkowski, Estrada, Milstead, & Hale, 1989; Case, Harris, & Graham, 1992; Montague & Bos, 1986; Slife et al., 1985), with elementary and junior high school students (Charles & Lester, 1984; Montague, 1991; Okamoto & Kitao, 1992), and with gifted and learning disabled students (Garofalo, 1993; Montague, 1992, 1993). However, relatively few studies actually investigated the cognitive and metacognitive framework for enhancing mathematical problem-solving skills (Artzt & Armour-Thomas, 1992; Lester, 1985; Montague, 1988, 1992). In this section, metacognitive theory and research on teaching mathematics problem-solving, especially with an emphasis on the underlying cognitive and metacognitive processes, are examined.

### Cognitive Processes

Considerable research on problem-solving has been based on Polya's four-step model: understanding the problem, developing a plan, carrying out

the plan, and looking back (Charles & Lester, 1984; Garofalo & Lester, 1985; Kersh & McDonald, 1991; Masingila & Moellwald, 1993; Taback, 1992). Polya's model has been useful in teaching mathematics problem-solving and improving students' performance in mathematics. Moreover, his model put an emphasis on heuristic thinking and reasoning at each phase of the problem (Case et al., 1992; Garofalo & Lester, 1985; McCoy, 1994; Silver & Marshall, 1990). Nonetheless, Polya's model has been criticized for being "clearly deficient as a description of actual problem-solving behavior" (Silver & Marshall, 1990). For this reason, many researchers elaborated on Polya's four-step model in order to organize mathematics problem-solving instruction for improving students' performance.

For example, Lester (1985) discussed four components of cognitive processes derived from Polya's model: orientation, organization, execution, and verification. Orientation refers to strategic behavior designed to arrive at an understanding of the problem. This includes representation and assessment of a task's familiarity, difficulty level, and the likelihood of arriving at a successful solution (e.g., "I try to understand the information and identify key points"). Organization involves the establishment of a solution plan, including identification of goals and subgoals (e.g., "I try to think of different ways to solve a problem"). Execution is the phase most typically associated with mathematics, that is, actually doing the computational work. Verification entails evaluation and monitoring of the three components of orientation, organization, and the execution of the

whole problem-solving process (e.g., "I make a table to help me to solve problems"). Verification also includes determining the adequacy of the actions performed, consistency of action with plans, consistency of local results with plans and problem conditions, and consistency of final results with problem conditions (e.g., "I check over my answers to avoid making mistakes").

Other researchers have also extended Polya's model. Schoenfeld (1985) specified five basic components: reading, analysis, exploration, planning/implementation, and verification. Bransford and Stein (1984) also conceptualized problem-solving processes using the acronym IDEAL: Identify the problem, Define and represent the problem, Explore possible strategies, Act on strategies, and Look back and evaluate the effects of your activity. Montague and Bos (1986) specified cognitive processes which consist of many components such as reading the problem, paraphrasing, visualizing, hypothesizing, estimating, computing, and checking.

As seen above, many researchers refined Polya's four-step model for mathematics problem-solving instruction. With respect to metacognition, Garofalo and Lester (1985) noted that Polya's four-step model lacks interaction between cognition and metacognition. Silver (1982) pointed out that many of Polya's heuristic suggestions are metacognitive prompts which can profoundly influence a problem solver's information-processing behavior. For example, when dividing 5 into 385, consider Polya's final stage, "look back over the solution," by multiplying 5 by 77. Zbiek (1989) raised a question: "Is

this a metacognitive effort to verify the solution process, or is it part of the cognitive strategy for doing long division?" It is often difficult to distinguish these two components (e.g., cognition and metacognition) (Keller & Lloyd, 1989; Zbiek, 1989). Nonetheless, an effective intervention program needs to be provided for students utilizing both cognitive and metacognitive process instruction to promote successful problem-solving. The next section reviews some of the research on metacognitive process instruction in mathematics problem-solving.

Metacognitive Processes

There is evidence that by making students conscious of their own thinking processes, learning can be enhanced. Many researchers contended that metacognition is crucial to success in problem-solving (Garofalo & Lester, 1985; McLeod, 1989; Romberg & Carpenter, 1986; Schoenfeld, 1987; Silver, 1982). In other words, by explicitly addressing metacognitive aspects of learning and problem-solving, students can be taught how to learn more effectively (Romberg & Carpenter, 1986). In most of the research on metacognition in mathematics problem-solving, the focus has been on metacognitive knowledge (Lester, 1985), self-awareness of strategies (Montague, 1988, 1991; Montague & Bos, 1986, 1990; Montague & Applegate, 1993), or accuracy of before- and after-task prediction (Slife et al., 1985) in order to identify characteristics of good problem solvers and improve students' performance in mathematics.

Lester (1985) conceptualized a metacognitive framework based on Flavell's notion of metacognitive knowledge (i.e., person, task, and strategy variables) which is prerequisite for higher-level learning and problem-solving. He believed that the individual's knowledge controls each of the cognitive components. He also believed that individuals with better metacognitive knowledge use executive procedures for better planning, execution, and monitoring of the problem-solving process. Similarly, in the study by Okamoto and Kitao (1992), children with high metacognitive knowledge appeared to have richer awareness of problem-solving than children with low metacognitive knowledge, particularly in the planning stage.

It would seem that having high metacognitive knowledge allows students to achieve higher success in their problem-solving tasks. Some studies have concluded that the students with high metacognitive knowledge are aware of their own thinking as it relates to personal strengths and weaknesses of motives and emotions, the task, and learning/thinking strategies (Cardelle-Elawar, 1995; Meltzer, 1984; Meltzer, Solomon, Fenton, & Levine, 1989). Moreover, they have control of their own thinking as it relates to independent, inter-dependent learning, and transfer. In other words, students with high metacognitive knowledge have an adequate knowledge base which must be integrated with knowing how to use the knowledge with widely applicable strategies. A considerable amount of learning from relevant experience forms the basis of problem-solving ability. Montague

(1991) examined gifted and LD gifted students' cognitive and metacognitive knowledge of mathematical problem-solving. Children were interviewed about their mathematics problem-solving during 2 one-hour sessions. During these sessions, they watched a videotape of the interviewer thinking aloud while solving a verbal reasoning problem and then observed a live demonstration of problem-solving and thinking aloud. Following the demonstration, they practised thinking aloud as they solved seven novel and text problems. It was found that gifted students without LD were strategic in their approach to problem-solving and also were aware of the strategies they use, whereas LD gifted students appeared to be relatively unaware of their knowledge, less refined in their approach to problem-solving, and less successful in their attempts.

In contrast to students with high metacognitive knowledge, LD students are frequently unaware of the usefulness of specific strategies for solving particular tasks, often displaying limited flexibility and inefficiency in accessing information at the appropriate times. Moreover, LD students do not recognize the presence of contradiction or disequilibrium which sets the stage for problem-solving and, in effect, have no need to monitor themselves because they are often not aware a problem exists (Cawley & Miller, 1986). In many of the studies conducted by Montague and her colleagues, LD students appeared to be less aware of their knowledge and use of problem-solving strategies than high and average achieving students (Montague & Applegate, 1993; Montague, 1991; Montague, & Bos, 1990). To provide an effective

intervention program for these students, cognitive and metacognitive process instruction needs to be considered essential for successful problem-solving. The next section reviews some of the research on cognitive and metacognitive process instruction in mathematics problem-solving.

## Strategies Instruction in Mathematical Problem-Solving

It has been suggested that teaching cognitive and metacognitive processes helps students promote generalized learning (Butterfield, 1994). Accordingly, many researchers have investigated the combination of teaching cognitive and metacognitive strategies as they relate to successful mathematics problem-solving. Nevertheless, it is not clear which strategies are most influential on students' performance on mathematics problem-solving.

For example, Slife et al., (1985) found that knowing how to solve problems is a different skill or ability from knowing that one knows how to solve problems. They investigated cognitive and metacognitive factors with 24 LD and 24 regular students at the elementary school level. With respect to cognitive factors, LD and regular students did not significantly differ on IQ scores, achievement in three different areas of mathematics (computation, application, and concepts), and performance on 10 arithmetic problems. They also measured metacognitive factors such as the accuracy of before-task prediction of correctness (i.e., knowledge of cognition), the accuracy of

identifying rights and wrongs (i.e., regulation of cognition), and the amount of time on identification task (i.e., regulation of cognition). It was found that the LD students were significantly less accurate both in predicting how many problems they could solve (i.e., knowledge of cognition) and in identifying correct and incorrect problems after their performance than were the regular students (i.e., regulation of cognition). Additionally, LD students were less sensitive in knowing about knowing (i.e., regulation of cognition). The results indicated that the two groups (LD and regular groups) differed on both components of metacognition: knowledge about cognition and regulation of cognition. It was concluded that cognitive factors could be separated from metacognitive factors. Results from Slife et al. seem to indicate that metacognitive factors may be more crucial for students' problem-solving skills than cognitive factors. The question then arises: Does teaching only metacognitive factors lead to improvement of students' performance in mathematics problem-solving? The answer, based on the research literature on cognitive and metacognitive strategies instruction, is "no." As will be described below, several studies indicated that combining cognitive and metacognitive instruction improved students' mathematics problem-solving skills (Cardelle-Ellawar, 1992, 1995; Coltharp, 1990; Montague, 1992; Montague & Bos, 1986).

Cardelle-Elawar (1995) reported significant positive effects of metacognitive strategies instruction on students' mathematics problem-solving ability. However, it is interesting to note that the metacognitive

instruction in this study was embedded in cognitive instruction. She investigated the effect of metacognitive instruction in mathematics instruction on low-achieving third to eighth grade students. Self-questioning was implemented as a metacognitive instruction. Lessons were presented in three stages (i.e., introductory discussion, independent work, and summary) for one year. In the first stage, introductory discussion, the teacher used the self-questioning process in front of students by asking and answering the four questions derived from Mayer's model (e.g., "Do I understand all the words in the problem?," "Do I have all the information to solve it?," "Do I know how I should organize the information?," "Do I know how to calculate the solution?"). In the second stage, independent work, the teacher provided individualized feedback while students worked individually. In the final stage, summary, two questions were frequently asked of the students; "What did you learn today?," "What did you learn about yourself in solving mathematics problems?" It should be noted, however, that metacognitive instruction was delivered in Mayer's model (i.e., translation, integration, planning/monitoring and solution execution) which put an emphasis on cognitive processes in mathematics problem-solving. Therefore, it should be considered that the positive results reported in her study were derived from both cognitive and metacognitive instruction rather than metacognitive instruction alone.

Montague (1992) found that either cognitive or metacognitive strategy instruction alone was insufficient for middle school LD students to improve

on the criterion level, whereas adding the complementary component of the cognitive and metacognitive strategy instruction package was, in general, successful. Similar findings were reported with adult subjects. In Coltharp's (1990) study, 39 university students were exposed to metacognitive strategies (i.e., planning, monitoring, and assessing) for 16 weeks. It was found that exposure to metacognition alone did not improve students' mathematical problem-solving ability. With regard to this insignificant effect, Simons (1994) asserted that teaching metacognitive skills to students who may already possess those skills is not likely to improve their mathematical skills. Moreover, for students who lack some basic cognitive skills, the positive effect of metacognitive instruction may not thereby occur (Simons, 1994). These results were corroborated by Montague and Bos (1986) who reported that teaching metacognitive processes alone to LD adolescents was not effective in successful mathematics problem-solving performance. They taught cognitive and metacognitive processes to 6 LD adolescents using two-step word problems and evaluated the results using a multiple-baseline design. Cognitive processes consisted of many components such as reading the problem, paraphrasing, visualizing, hypothesizing, estimating, computing, and checking. Metacognitive processes incorporated components such as self-questioning, self-instruction, and self-monitoring. Montague and Bos found that training which included both cognitive and metacognitive strategies had more beneficial effects on students' performance than training which only included either cognitive or metacognitive strategies.

As reviewed above, cognitive and metacognitive strategies instruction (Montague, 1992; Montague & Bos, 1986; Cardelle-Elawar, 1995) seems to be pivotal to enhance students' mathematics problem-solving skills since cognition and metacognition are related to each other (Butterfield, 1994). Montague (1988) and Lester (1985) pointed out that there is an interaction between the cognitive and metacognitive processes in mathematical problem-solving (Figure 1). Artzt and Armour-Thomas (1992) also mentioned that appropriate interaction between these two components is necessary for successful problem-solving to occur. Therefore, as noted by Kilpatrick (1985), mathematics problem-solving is better understood as *recursive and reflective activities* requiring cognitive and metacognitive strategies (Figure 2).

In order to study the interaction between cognitive and metacognitive processes, Arzt and Armour-Thomas (1992) examined problem-solving behaviors of 27 mathematically average-ability students who worked in small groups. These researchers built a framework based on heuristics and cognitive and metacognitive levels of problem-solving. Students in grade seven were videotaped while working in small groups to solve a mathematical problem. Based on the videotapes, the students' behavior was categorized in two ways: by the episodes which were derived from Schoenfeld's (1985) model (e.g., reading, analysis, exploration, planning/ implementation, and verification), and by cognitive level which was categorized by Lester (1985) and Flavell (1981). More specifically, *reading* was categorized as predominantly cognitive, while *understanding, analyzing, and*

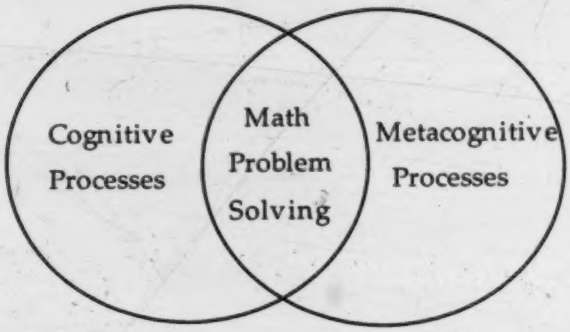



Figure 1. Interaction between the cognitive and metacognitive processes

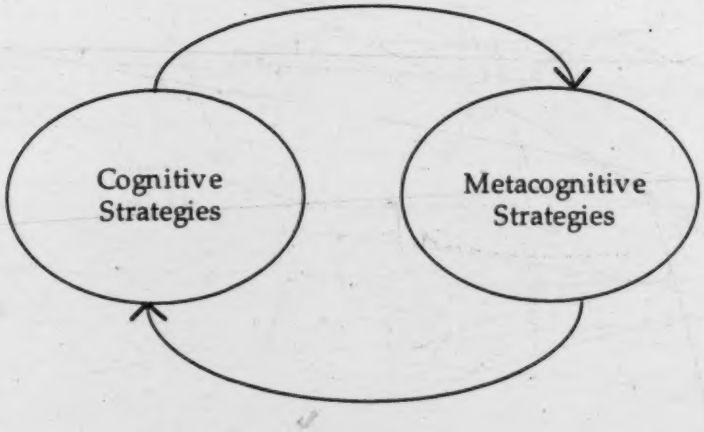



Figure 2. Recursive and reflective activities of mathematical problem-solving

*planning* were categorized as metacognitive. *Exploring, implementing, and verifying* were categorized as both cognitive and metacognitive. "Cognition is implicit in any metacognitive activity, and metacognition may be present during a cognitive act, although perhaps not apparent" (Arzt & Armour-Thomas, 1992, p. 141). Arzt and Armour-Thomas (1992) found that, of 442 behaviors coded, 38.7% were metacognitive, 36.0% were cognitive, and 25.3% were in the "watch and listen" category, that is, an undetermined cognitive level. Of 171 behaviors coded as metacognitive, the majority were in the exploring (36.3%) and understanding (32.2%) categories. Of 151 behaviors coded cognitive, 60.4% were in the exploring category, and 23.9% were in the reading category. Exploring (cognitive and metacognitive) was the behavior that was categorized the greatest percentage of times. They also found that of all the groups, the group which did not solve the problem appeared to have the lowest percentage of metacognitive behaviors of all the groups.

In contrast to Slife et al., (1985), Arzt and Armour-Thomas (1992) failed to show a clear distinction between cognition and metacognition in problem-solving. In the area of mathematics word problem-solving, several researchers, along with Arzt and Armour-Thomas, have categorized two of the steps (i.e., checking and verification) as cognitive processes; however, they also found that these steps resemble metacognitive processes in some aspects (Garofalo & Lester, 1985; Lester, 1985; Montague, 1991; Montague & Bos, 1986). These researchers confirmed that cognitive measures require metacognitive

skills to some degree. This was also supported by Nandi's (1993) study on intellectually gifted and average children showing that cognitive and metacognitive functioning (r = .42 - .55) partially overlap each other.

In many research studies, metacognition has been defined as knowing what one is doing. These studies have shown that teaching students to be aware of what they are doing may be the way to enhance their metacognitive processes. Cognitive and metacognitive process instruction may be possible using computers as tools: "Computer tools can afford the activation of higher order mental operations by relieving learners of low-level, tedious operations and heavy reliance on memory capacity" (Salomon, Globerson, & Guterman, 1989, p. 621). In the following section, the potential of applying computer-based instruction (CBI) to cognitive and metacognitive strategies instruction is explored.

## Using Computers to Foster Strategies Instruction

Strategies are useful techniques, principles, or rules for solving widely dissimilar problems of varying degrees of difficulty and complexity; as well, they are not problem-specific. They represent procedures which learners can use to make a problem clearer, simpler, smaller, or more manageable (Suydam, 1978). In short, strategies instruction empowers students to become actively engaged in the learning process, to control their thinking, to make decisions, to take actions, and to become successful and independent learners (Day & Elsknin, 1994; Van Reusen & Head, 1994). Moreover, by incorporating

strategies instruction into the curriculum, passive and resistant learners become motivated and self-reliant learners (Lock & Abbey, 1989). For example, Schoenfeld (1985) found that learning strategies[3] (i.e., subskills) significantly increased students' problem-solving abilities. In his study, seven college students who were from advanced courses in mathematics were given a set of problems which could be solved using strategies. Four students in the experimental group received explicit strategies training for five individual sessions over a period of two weeks, while three students in the control group did not. The results were that explicit instruction in specific and general problem-solving methods was efficient and has clear and rapid effects, not only on problem-solving abilities, but also on students' beliefs regarding these abilities.

As with Schoenfeld, Hembree (1992), in a meta-analysis using 487 studies on mathematics problem-solving which appeared in the literature from the 1920s to 1990, also found that explicit training was essential to develop subskills. Without direct intervention, practice in using learning strategies did not result in better performance (Hembree, 1992). In addition, training to handle a wide variety of problems over time seems to be essential (Lester, 1983, 1985). Modeling master problem solvers can be useful, as noted by Bloom and Broder (1950), because students can compare their solutions to

---

[3] Leaning strategies refer to "techniques, principles, or rules that will facilitate the acquisition, manipulation, integration, storage, and retrieval of information across situations and settings" (p. 13).(see Alley & Deshler, 1979).

those of a model student (i.e., one who has mastered problem-solving techniques).

As described above, many studies support the evidence that problem-solving instruction should involve explicit training on both specific and general strategies. Furthermore, providing opportunities to practise strategies seems to be essential for enhancing students' mathematics problem-solving skills.

The ultimate goal of the strategies approach is to directly teach thinking skills. This can be done in any subject by teaching process as well as content. Some researchers have suggested that computers can help to improve the learning process (Cardelle-Elawar & Wetzel, 1995; Tringa & Lipitakis, 1995) and can be used to teach problem-solving skills explicitly (Silver, 1985). Combining computer-based instruction (CBI) with structured teaching can be successfully used to help students develop their thinking skills, and therefore can be a powerful supplement to classroom instruction (Bennett, 1992; Hofmeister, 1990; Woodward, Carnine, & Gersten, 1988).

First, computers make it possible to give more personal attention to individual students resulting in an increase in motivation (Baldrey, 1991; Funkhouser, 1993; Geisert & Futrell, 1995; Okolo, 1992). Baldrey (1991) suggested that this motivational attribute is achieved through students' experience of self-directed behavior and by the use of the computer graphics and sounds which draw their attention to the task. This was also supported by Funkhouser's (1993) study which found that an increasing number of

students who used problem-solving software showed positive attitudes toward mathematics and performed better in geometry and algebra than those with non-computer augmented instruction. Furthermore, Okolo's (1992) study of the impact of attribution retraining, embedded within a mathematics computer-assisted instructional program, showed that the computer can be a useful tool for improving achievement motivation as well as academic skills of students with learning disabilities.

Second, computer-based learning environments can serve the function of easily providing the processes and knowledge that students frequently require for expertise in the subject area. Computers can be readily used to present facts repeatedly and tirelessly in interesting formats. Furthermore, computers can provide the results of the student's work in a few seconds, as well as rapid remedial help if the student is having difficulties (Baldrey, 1991). These features can prevent the teacher from becoming impatient or frustrated. This lowering of tension or social stress may be especially important when interacting with a student who is inclined to emotional or aggressive outbursts (Baldrey, 1991). There is evidence that the frequency of problem presentation (i.e., knowledge) increases the likelihood of retrieval (Siegler & Shrager, 1984) which in turn promotes the automaticity of mathematics skills (Lin, Podell, & Tournaki-Rein, 1994). More specifically, repeated exposure to basic arithmetic facts and to how strategies can be applied in problem-solving situations can be developed in CBI. This is because in the context of a full classroom in which the teacher has to manage thirty or more

students and with scarce resources, it may be difficult to provide repeated exposure to children, especially those with learning difficulties. In Lin, Podell, and Tournaki-Rein's (1994) study, the impact of computer-assisted instruction (CAI) using the software *Math Blaster* (Davidson, 1986) was examined by enhancing automaticity of mathematics skills with 140 mildly mentally handicapped and nonhandicapped students with a mean chronological age of 7.82. CAI, which is also known as CBI, sessions were 15 minutes per session for 13 to 16 sessions, depending upon the length of time needed to attain mastery of all 13 lessons. They found that computers can be used to promote automaticity of mathematical skills, especially for students with mild mental handicaps.

Finally, as reported in several research studies, computers can be used to teach strategies explicitly in order to improve students' problem-solving skills (Cardelle-Elawar & Wetzel, 1995; Hofmeister, 1990; Jussel, 1993; Woodward, Carnine & Gersten, 1988). Cardelle-Elawar and Wetzel (1995) used instructional game software developed by Wings for Learning/Sunburst Company with 120 second- and fourth-grade students for one hour per week over 15 weeks. In their study, students working in pairs with the computer were instructed by their teachers to use a self-regulatory strategy, "IDEA model (Cardelle-Elawar, 1993) to identify, define, explore strategies, and assess the progress in solving problems" (p. 388). They found that through using the computer as a mediator of problem-solving, self-regulated strategies helped students become self-regulated learners. However, since there was no

control group involved in their study, it is uncertain whether their positive findings were affected by the computer activities or the self-regulatory strategy. Unlike the Cardelle-Elawar and Wetzel (1995) study, Woodward, et al. (1988) included a control group to investigate the effectiveness of computer simulation (*Health Ways*) with 30 learning disabled high school students for 40 minutes per day for 12 days. These students were randomly assigned to one of two instructional groups (i.e., structured teaching and a combination of structured teaching and a computer simulation). They found that the simulation group (i.e., experimental group) performed at a significantly higher level on problem-solving skills in health.

Despite these positive findings from these studies described above, strategy instruction was not embedded in the computer programs. This may be because embedded strategy instruction via computer requires more effort and consideration in computer-based instruction (CBI) design as well as being immensely time-consuming.

Subsequently, little research has been done related to computer use for teaching mathematics problem-solving, even though the National Council of Teachers of Mathematics (NCTM) has strongly suggested technology use at all levels since 1980. It is surprising to find that for the last fifteen years, less than 40 studies on computer use for teaching mathematics problem-solving have appeared in major journals dealing exclusively with mathematics education such as Journal for Research in Mathematics Education and Journal of Computers in Mathematics and Science Teaching. This is reflected in

Ralston's (1993) comments: "Mathematicians.... have been very, very slow to adopt the use of technology as part of their teaching arsenal. Chalk and chalkboard are still the technology of choice for most mathematicians, with the more adventurous ones opting for overhead projector and screen" (p. 81). It should be pointed out that a lack of technology-related research in mathematics reflects the mathematics community's lack of technological engagement in education research. This is because technology-related research has been influenced heavily by practical complications such as getting subjects and computers in the same location at the same time, and providing teachers and students with appropriate computer and curricular materials (Kaput & Thomson, 1994). As well, computer materials are very expensive, require a considerable amount of time to produce, and are seldom available in the needed form.

In order to improve the quality of the computer-aided instructional materials, they should be user-friendly and designed in concert with the curriculum. Adapting computers to classroom instruction with focus being placed on developing higher-order thinking skills (i.e., mathematics problem-solving skills) is a key challenge for teaching and learning in the 21st century (Cardelle-Elawar & Wetzel, 1995). Even though the development of programs of this type is enormously time consuming, good instructional design is urgently needed since a good computer-aided environment in the classroom will enable teachers to use computers as their assistants in teaching mathematics as an attractive and dynamic subject (Ralston, 1993). Moreover,

this may enable students to become more efficient, thoughtful, independent, and reflective learners.

# III. Rationale and Research Questions

## Rationale

"How does problem-solving ability develop? ... How can students be helped to become better problem solvers and, hence, better doers of mathematics?" (Lester, 1994, p. 672) These questions have been a major concern in mathematics education over the past twenty years; however, they could be addressed by mathematics problem-solving strategies instruction. In the National Council of Teachers of Mathematics (NCTM) (1989) report, mathematics education reformers called for educators to embrace problem-solving as the basis for mathematics instruction. The more recently published A Perspective on Reform in Mathematics and Science Education recommended that through strategy use, students would be able to (a) investigate and understand mathematical content, (b) recognize and formulate problems, (c) solve a wide variety of problems, (d) generalize solutions and strategies, applying them to new problems, and (e) acquire confidence in their ability to use mathematics meaningfully (NCTM, 1996).

Some research has reported that teaching both cognitive and metacognitive strategies has a powerful effect on students' performance in mathematics problem-solving (Montague & Bos, 1986; Montague, 1992). This effect seems to be by virtue of the interaction between the cognitive and metacognitive processes in mathematics problem-solving (Arzt & Armour-Thomas, 1992; Lester, 1985; Montague, 1988). Moreover, many researchers

have pointed out that, in order to enhance students' mathematics problem-solving skills, strategies should be taught more explicitly and specifically (Hembree, 1992; Lester, 1985; Schoenfeld, 1985).

In Hembree's (1992) meta-analysis using 487 studies of mathematics problem-solving, he found most research done for the last seventy-five years is mainly in three areas: "(1) instruction in using a *method*, that is, a set of boundary procedures; (2) instruction in subskills of problem-solving (e.g., drawing figures or writing equations); and (3) instruction in related fields (e.g., computer programming)" (p. 261). As might be expected, it seems that there is still a relative lack of research on the combination of specific and general strategies to enhance cognitive and metacognitive processes in mathematics problem-solving skills.

With respect to problem-solving instruction, some research has supported the use of the computer as a tool to teach explicit problem-solving skills (Cardelle-Elawar & Wetzel, 1995; Dudley-Marling & Owston, 1988; Silver, 1985). This is because the computer can provide well-structured, individualized, and user-friendly instructional presentations which facilitate learning. In addition, considerable findings in the past have indicated that the computer can serve as a highly effective mode of delivery for instruction in a wide variety of instructional settings (Kulik, 1994). However, in spite of the availability of this technology, most research on mathematics problem-solving has been conducted through teacher-directed instruction in the classroom setting rather than through computer-based instruction in the

schools. Moreover, there is a paucity of literature on the effect of strategies instruction in mathematics problem-solving using a computer-based instruction (CBI) program.

Thus, this exploratory study compared the effects of using two types of CBI programs (with and without explicit mathematics problem-solving strategies). The experimental group received explicit strategies embedded within the CBI program and the corresponding CBI control group received the identical program, but did not receive explicit strategies instruction. For the traditional control group, students were not given any type of CBI program.

The purpose of this study was to explore the potential effects of explicit strategies instruction for sixth-grade students on mathematics problem-solving achievement, self-prediction of performance (SPP), self-evaluation of performance (SEP), accuracy of self-prediction of performance (ASPP), accuracy of self-evaluation of performance (ASEP), and the perception of mathematics problem-solving processes (MPSPQ). Moreover, attitudes toward mathematics, mathematics problem-solving, and the CBI programs were examined.

## Research Questions

The following questions were examined in this study:

1. Are there significant mean score differences on mathematics problem-solving performance for achievement groups (high and low) among three groups (experimental, CBI control, and traditional control)?
2. Are there significant mean score differences on SPP for achievement groups (high and low) among the three groups (experimental, CBI control, and traditional control)?
3. Are there significant mean score differences on SEP for achievement groups (high and low) among the three groups (experimental, CBI control, and traditional control)?
4. Are there significant mean score differences on knowledge about cognition (i.e., ASPP) for achievement groups (high and low) among the three groups (experimental, CBI control, and traditional control)?
5. Are there significant mean score differences on regulation of cognition (i.e., ASEP) for achievement groups (high and low) among the three groups (experimental, CBI control, and traditional control)?
6. Are there significant mean score differences on the perception of mathematics problem-solving processes for achievement groups

(high and low) among the three groups (experimental, CBI control, and traditional control)?

7. Are there significant differences on attitudes toward mathematics, mathematics problem-solving, and computer use among the three groups (experimental, CBI control, and traditional control)?
8. Is there a significant difference between the experimental and CBI control groups on their attitudes toward the CBI program which they received?

# IV. Method

This chapter describes the method used in the study. It contains a description of the research design, subjects, instructional design and development, instrumentation, procedure, and data analysis.

## Research Design

A pre-test--post-test quasi-experimental control group design was implemented to conduct this study. This type of design was chosen because the experimental, computer-based instruction (CBI) control, and traditional control groups were such naturally assembled groups as intact classes. Random assignment of participating schools to three groups was impossible.

The overall time involved in collecting data was eight weeks, with three weeks as the pre-test phase, three weeks as the CBI program intervention phase, and two weeks as the post-test phase. The time period involved in this study was September through November 1997.

There was one experimental group and two control groups (CBI control, and traditional control). The experimental group was selected from three intact classes of sixth-grade students at a suburban elementary school. The Problem-Solving Strategies (PSS)-CBI Program was installed on the hard drives of twenty-two workstations in the lab at the elementary school. Each workstation consisted of a Macintosh LC575 and a 14 inch color RGB display.

Approximately 90 minutes per week were allocated for the explicit strategy instruction via computer for three weeks during October 1997.

The CBI control group was selected from two intact classes of the sixth-grade students in another elementary school in the same school district. Each workstation for the CBI control group consisted of a Macintosh 5400 Series and a 14-inch color RGB display. Since the explicit strategies were not embedded in the program for the CBI control group, they only spent approximately one 60 minute session per week for the mathematics problem-solving instruction via computer (Control-CBI program) for the three weeks during October 1997. Except for the strategy instruction for the experimental group, all the contents in the computer programs for both groups were identical. All the students in the experimental and CBI control groups worked under the supervision of the researcher and the classroom teacher.

The traditional control group was selected from three intact classes of the sixth-grade students in other elementary schools in the city and suburban area. These students did not receive any type of CBI program during this study.

The independent variables for this study were the mathematics problem-solving achievement level (high and low) and the instructional type (experimental, CBI control, and traditional control). The dependent variables were the mathematics problem-solving sub-test from the Canadian Test of Basic Skills (CTBS-MPS), the Mathematics Problem-solving Process Questionnaire (MPSPQ), the Mathematics Problem-solving Test (MPST), and

the Attitude survey. The dependent variables were measured during the pre-test and post-test periods to ascertain if performance improvement was directly related to the explicit strategy instruction via computer.

## Selection of Subjects

The subjects were 186 sixth-grade students, aged 11 and 12 years, who were drawn from six sixth-grade and three mixed-grade classrooms in five public elementary schools. Two schools in Edmonton, one in Sherwood Park, and two in Fort Saskatchewan took part in the study during the fall of 1997. The schools varied in classroom size and computer laboratory facilities. Two schools did not have a sufficient number of Macintosh computers to administer the CBI program. Therefore, teachers at these schools were asked to be involved in the traditional control group. School teachers at each of three schools with Macintosh computer laboratories were asked to choose one of the treatment conditions (experimental, CBI control, or traditional control). The control group schools were provided with the CBI program after they completed the post-test phase. All subjects were assigned by schools to three groups (experimental, CBI control, and traditional control). These groups included three intact classrooms for the experimental group, two intact classrooms for the CBI control group, and four intact classrooms for the traditional control group. Of the 186 sixth-grade students, the parental or legal guardian and student consent forms were obtained from 182, resulting in a 98% consent rate. Of these students, 94 students (52%) were male and 88

students (48%) were female at the beginning of the study. However, due to a variety of factors such as excessive absences, moving out of the area, and incomplete tests, only 115 students (60 males and 55 females) were assessed on their mathematics problem-solving achievement levels (high and low). For this measure, students' raw scores from a pre-test which consisted of a mathematics problem-solving sub-test from the CTBS (CTBS-MPS) were used to ensure there was equal distribution of students' mathematics problem-solving achievement to the greatest extent possible within the limitations of an experimentally accessible population.

It was found that the CTBS-MPS scores ranged from 5 (the 5th national percentile) to 29 (the 99th national percentile) with a mean of 16.64 and standard deviation of 6.23. The median score for the CTBS-MPS was 16. Students whose median scores were above 16 (the 71st national percentile) were identified as belonging to the high achieving group. Students whose median scores fell below 16 (the 63rd national percentile) were identified as belonging to the low achieving group. Ten students whose median scores were at 16 were excluded from data analyses. As shown in Table 1, this resulted in the experimental group consisting of 39 students and the CBI control group consisting of 29 students, while the traditional control group had 37 students. It was observed that there were no significant differences among the three groups on the pre-CTBS-MPS, $F(2, 102) = .91$, $p > .05$. This indicated that students were homogeneous for mathematics problem-solving achievement. The mean scores of the national percentile rank on the

mathematics problem-solving achievement for the experimental, CBI control, and traditional control groups were the 67th, 71st, and 71st percentiles, respectively. Equal samples for the three groups were not possible.

Table 1

Sample Size for Each Group

| | Experimental | CBI Control | Traditional Control | Total |
|---|---|---|---|---|
| High | 19 | 18 | 20 | 57 |
| Low | 20 | 11 | 17 | 48 |
| Total | 39 | 29 | 37 | 105 |

## Instructional Design and Development

The computer-based instructional design was based on ideas in SPELT -- *A Strategies Program for Effective Learning and Thinking* (Mulcahy, Marfo, Peat, & Andrews, 1987). The SPELT approach emphasizes the progression from the lowest level of strategy acquisition (teacher-imposed) to the highest level of acquisition (student self-generation). The essence of this approach is to nurture the students' learning and motivation through "effective and innovative methods" such as direct instruction of learning strategies, peer tutoring, Socratic dialogues, and cooperative learning. This approach involves three phases of instruction: direct teaching of cognitive and

metacognitive strategies, systematic transfer of the strategies, and generalization of the strategies through more interactive and reflective teaching. A major focus of SPELT is to make students more aware of the benefits of cognitive and metacognitive strategies, to teach them to utilize strategies, and then to self-generate effective learning and thinking strategies (Marfo, Mulcahy, Peat, Andrews, & Cho, 1991; Mulcahy, 1991). The results of using the SPELT approach were very positive "both from the perspective of student change as well as from the perspective of the teachers, parents, and administrators" (Mulcahy, 1991, p. 390) in a three-year longitudinal study. Hence, it is expected that adapting this approach to mathematics problem-solving via computer may help students improve their mathematical understanding, facilitate their performance, and also promote favorable attitudes toward mathematics.

For the experimental and CBI control groups, two different versions of the CBI program were developed using an authoring software, Authorware Professional Version 3.5. These versions were used on the Macintosh computer equipped with a color monitor. Graphics and animation to illustrate content in the program were developed using ClipArt Library, Microsoft ClipArt Gallery, Aldus SuperPaint 3.5, and Authorware Professional Version 3.5. As shown on the Project Timeline in Table 2, the total time for developing the two versions of the CBI program for this study took an extensive amount of time (i.e., one and a half years). The first stage of CBI program development was Planning and Design which took over four

months (January 1996 to first week of June 1996). The second stage was Lesson Programming which took six months (mid-June 1996 to mid-January 1997). The third stage was Evaluation with Content Experts which lasted approximately two months (March 1997 to April 1997). The fourth stage was Refinement of Program which was two and a half months in length (April 1997 to mid-June 1997). The fifth stage was Evaluation with Students which took one and a half months (early June 1997 to mid-July 1997). The sixth stage was Final Evaluation with Experts and Final Refinement of Program which took about two months (July 1997 to early September). The last stage was Implementation of Program which took approximately five weeks (last week of September 1997 to end of October 1997).

Table 2

Project Timeline

| | 1996 | | | 1997 | | | |
|---|---|---|---|---|---|---|---|
| | Jan | June | Dec | Jan | June | Sep | Oct |
| Planning and Design | | | | | | | |
| Lesson Programming | | | | | | | |
| Evaluation with Content Experts | | | | | | | |
| Refinement of Program | | | | | | | |
| Evaluation with Students | | | | | | | |
| Final Evaluation with Experts and Final Refinement of Program | | | | | | | |
| Implementation | | | | | | | |

These versions were a CBI program with strategies instruction for the experimental group and a CBI program without strategies instruction for the CBI control group. Both versions presented identical mathematics word problems. However, the experimental group received general and specific problem-solving strategies (see Figure 3), whereas the CBI control group participated in three CBI lessons without instruction on general and specific problem-solving strategies (see Figure 4). The two versions of the CBI program for the experimental and CBI control groups were developed by the researcher and reviewed by experts in the fields of mathematics and strategies instruction to ensure the accuracy of the program contents and design. Both CBI programs incorporated three basic components in all lessons: practice, quiz, and result. The CBI programs for the experimental (PSS-CBI) and CBI control (Control-CBI) groups are described below.

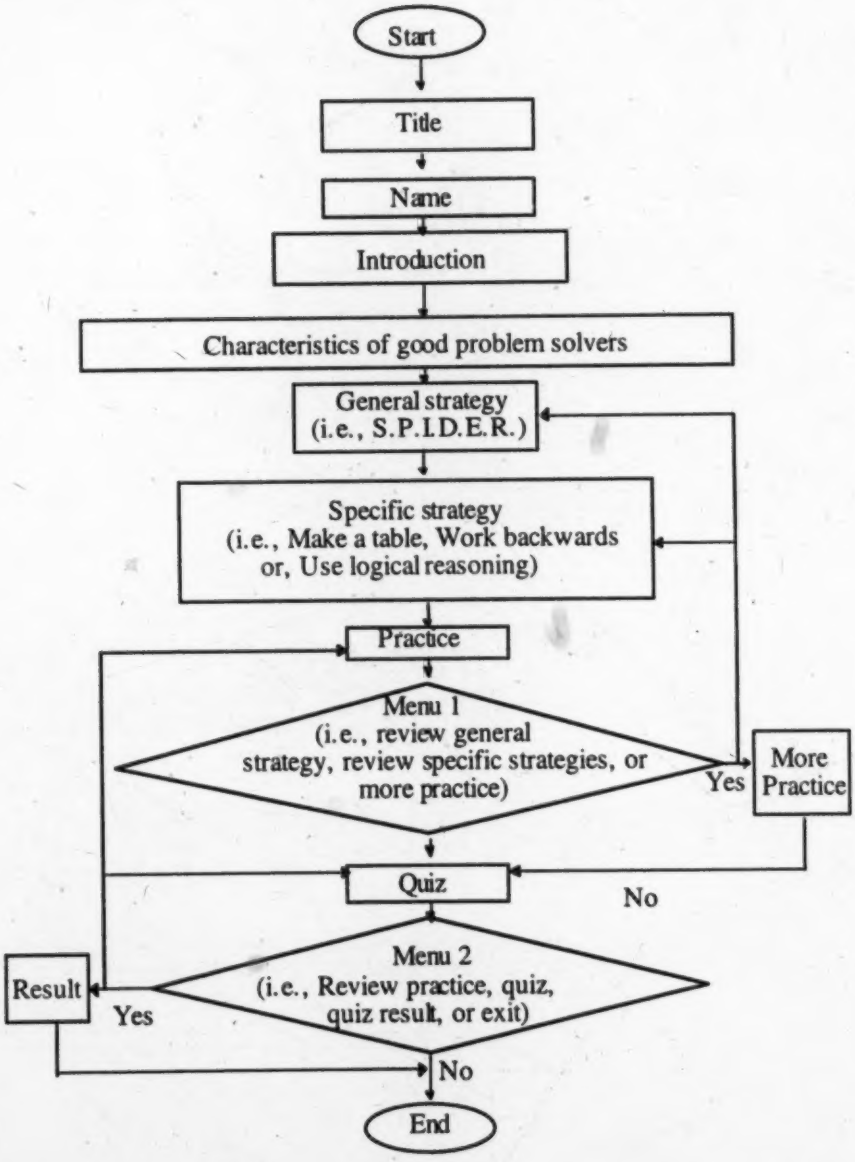



Figure 3 . Flowchart for Problem-Solving Strategy (PSS)-CBI program (experimental group).

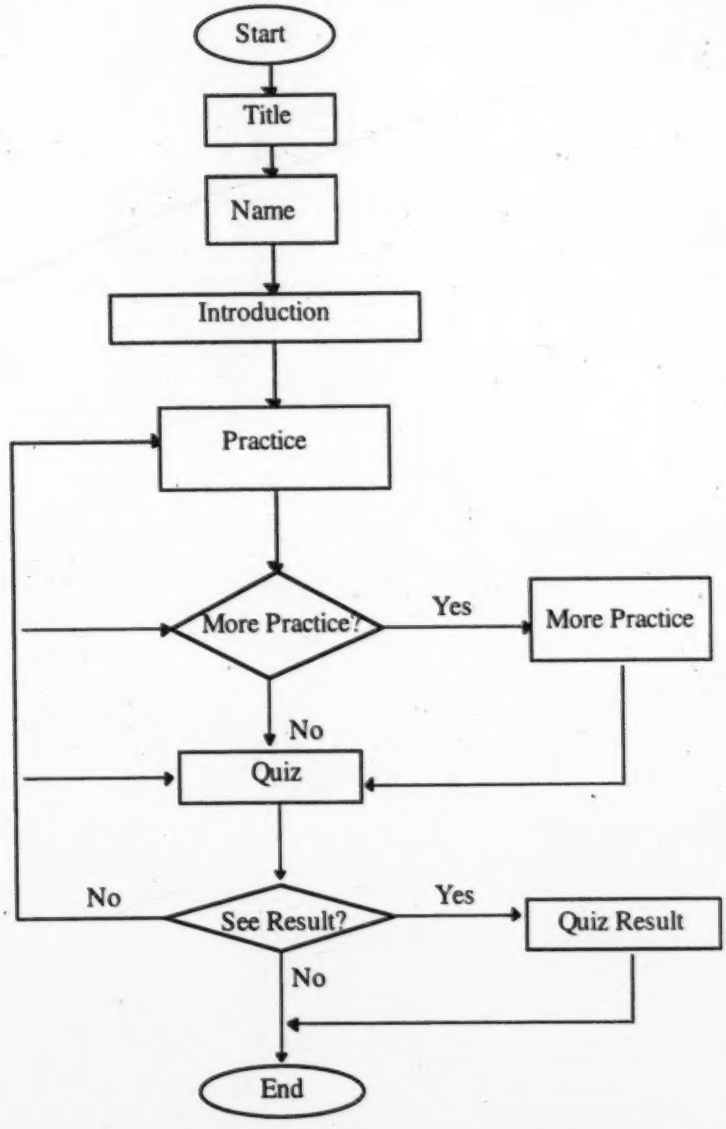



Figure 4. Flowchart for CBI-Control program (CBI Control group).

Problem-Solving Strategies Embedded CBI Program (PSS-CBI)

The PSS-CBI program consisted of three lessons. Each lesson itself consisted of five sections: (1) introduction (2) demonstration of strategies (general strategy: SPIDER, and specific strategies: Make a table, Work backwards, and Use logical reasoning), (3) practice of strategies (Practice), (4) assessment of skills and strategies (Quiz), and (5) results.

Students were required to type their names and the name of their school at the beginning of each lesson. After students logged in, a main menu screen was presented followed by operational instructions (i.e., How to use main features of the lesson) (see Figures 5 to 7).

Screen design. In order to facilitate lesson navigation, the main menu screen was divided into four major areas: informational, directional, functional, and navigational areas (see Figure 8).

An informational area provided the lesson content in the central part of the screen. Graphics and color were used to visually illustrate the contents, to hold the student's attention, and to enhance the student's learning processes.

A directional area presented six activities at the bottom of the screen. These activities were text-based buttons labeled Introduction, General Strategy (SPIDER), a Specific Strategy (e.g., Making a table, Working backwards, or Using logical reasoning), Practice, Quiz, and Result. Students were asked to

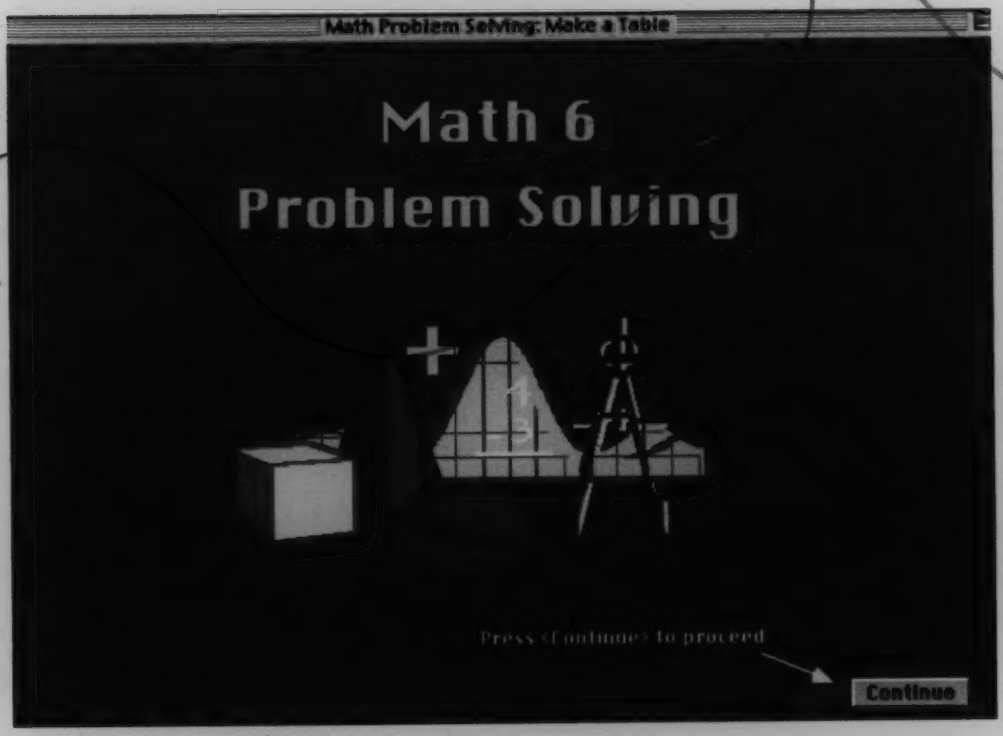



Figure 5. Main Title Screen for CBI program

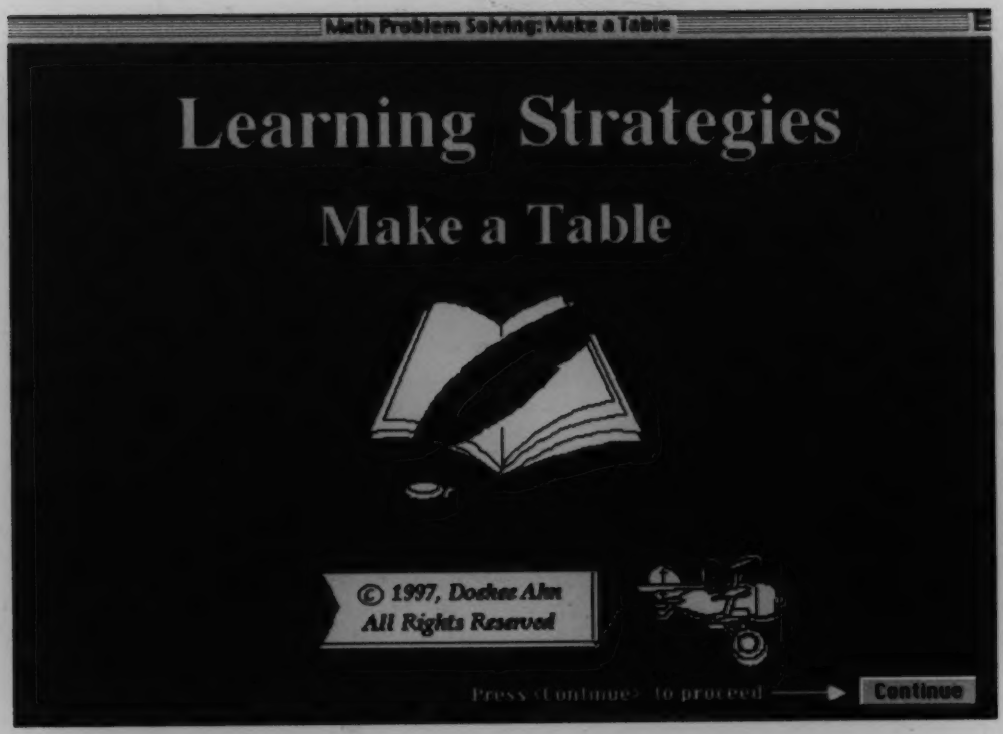



Figure 6. Specific Title Screen for Learning Strategies Component

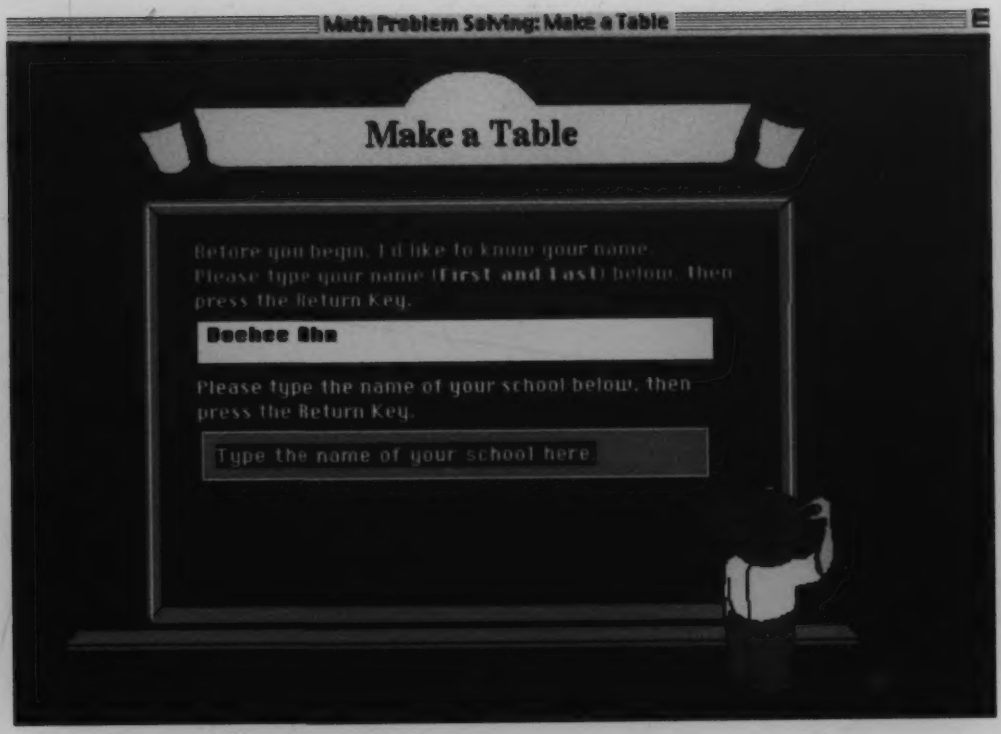

Math Problem Solving: Make a Table
Make a Table
Before you begin, I'd like to know your name.
Please type your name (First and Last) below, then
press the Return Key.
Duchee Rha
Please type the name of your school below, then
press the Return Key.
Type the name of your school here

Figure 7. Login Screen

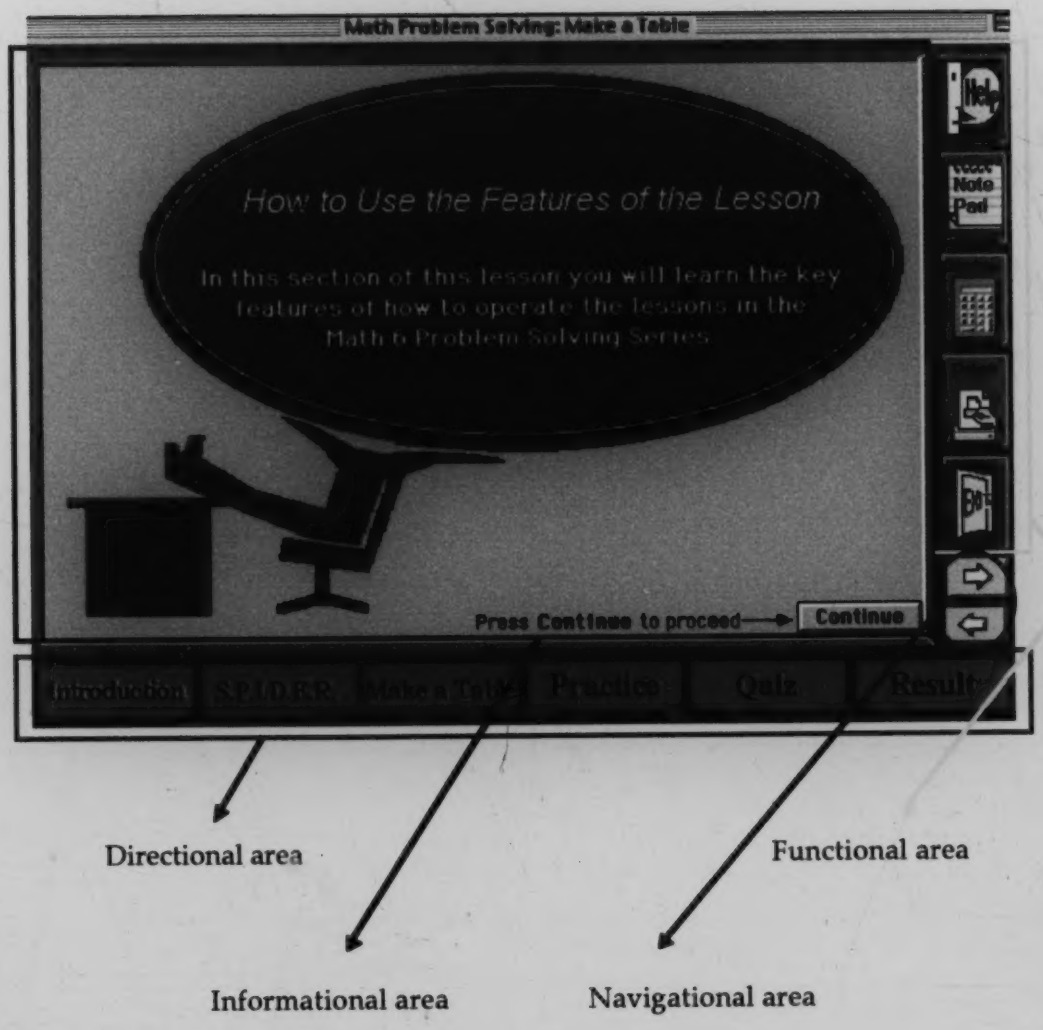

Math Problem Solving: Make a Table
How to Use the Features of the Lesson
In this section of this lesson you will learn the key features of how to operate the lessons in the Math to Problem Solving Series.
Press Continue to proceed
Continue
Introduction
SPIDER
Make a Table
Practice
Quiz
Results
Directional area
Functional area
Informational area
Navigational area

Figure 8. Screen Design

choose the activities in sequential order. During each activity, a sub-activity bar was available to remind the student of the current time, topic, and page. This feature was designed to allow the student to control the time as well as the speed of presentation.

A functional area presented five functions on the right side of the screen. These functions, which were graphic-based buttons, included: 1. Help, 2. Note Pad, 3. Print, 4. Calculator, and 5. Exit (see Figure 8). The buttons in this area always appeared on the screen, so they were accessible to the student at any point in the program with ease. Also, these features allowed the student to return to where he or she left off without any difficulty. The "Help" button allowed the student to learn the key features of how to operate the lesson whenever it was activated. The "Note Pad" button allowed the student to write a note whenever he or she wished. This function provided opportunities for the student to retrieve and update information without any difficulty. The "Print" button allowed the student to print the contents of the screen. The "Exit" button was designed to allow the student to exit from almost any point in the program. This feature also offered an error correction function by asking the student whether he or she really wanted to exit the program. If students decided not to quit the program, they were able to go back to the point at which they left off.

A navigational area presented three functions at the bottom of the right side of the screen in the form of the following buttons: 1. left-pointed arrow (←), 2. right-pointed arrow (→), and 3. Main Menu. The "Main Menu"

button allowed the student to go back to the main menu whenever he or she wished. This function was available when the student reached either the end of the introduction, the SPIDER, the specific strategy, or at the end of each practice question. To accommodate the student's reading rate and reading speed, the left-pointed arrow and the right-pointed arrow buttons were included in the navigational area. The left-pointed arrow button allowed the student to return to the previous screen to review by either simply clicking on the left-pointed arrow in the lower right corner, or pressing the left-pointed arrow key on the keyboard. This function was available during the Introduction, the General Strategy (SPIDER), and the Specific Strategy (e.g., Make a table, Work backwards, or Use logical reasoning) sessions. The right-pointed arrow button allowed the student to proceed through the lesson at his or her own speed by either clicking on the right arrow in the lower right corner of the screen or pressing the return key or Right Arrow key on the keyboard. These buttons were designed to allow the student enough time to read and understand the contents on the screen.

In order to enhance interaction between students and the program, students received two or three chances to answer the question when they made a response error. They were also given immediate feedback about their responses to the mathematics word problems. Individual feedback provided the student with an evaluation of the correctness of the response, as well as a detailed explanation regarding the student's error.

Lesson contents. The PSS-CBI program was designed to teach the benefits of learning strategies (i.e., general and specific strategies) and to demonstrate how and when to use them. The structure of the CBI program consisted of three lessons as follows:

Lesson One: Make a table

Lesson Two: Work backwards

Lesson Three: Use logical reasoning

Each lesson for the experimental group began with a tutorial that modeled problem-solving strategies (i.e., subskills). Students were taught how to solve mathematics word problems with a general strategy that has been adapted from Polya (1957). Each lesson was constructed as follows: (1) introduction of the lesson (e.g., determining the appropriate strategy for solving a problem), (2) description of a general strategy, *SPIDER* , -- (a) Start to read the problem, (b) Point out what the question is, (c) Identify things you need to know, (d) Decide upon a type of strategy you plan to use, (e) Execute your strategy to solve the problem, (f) Remember to check your solution -- derived and modified from Polya (1957)'s four step model for problem-solving, (3) instruction of a specific strategy (e.g., make a table, work backwards, or use logical reasoning), (4) practice of mathematics word problem-solving through the use of strategies, (5) more practice of mathematics word problem-solving (optional), or review of any of (1), (2), (3),

or (4), (6) quiz, (7) results of the quiz, (8) review of the lesson (optional), and (9) exit (see Figures 3 and 8). These features are described below.

Introduction. During the introduction, students were informed of what they were going to learn in the lesson. They were also encouraged to study mathematics problem-solving by being provided with positive feedback on their response.

SPIDER. A good deal of research in the last decade has shown that mnemonic techniques are effective strategies for enhancing learning. From a variety of mnemonic techniques (e.g., keyword method, pegword method, reconstructive elaborations, etc.), acronym mnemonics are the most applicable for enhancing the acquisition of basic facts and word problem-solving skills in mathematics (Miller & Mercer, 1993). Therefore, it was decided to use an acronym mnemonic technique in the general strategy session to help students recall problem-solving components.

In the general strategy (SPIDER) session, students were given descriptions of good problem solvers and things they need to do to solve problems in a structured step-by-step manner. In order to help students remember the steps in problem-solving process, a first-letter mnemonic device was developed by the researcher. This resulted in a six-step strategy called "SPIDER":

1. Start to read the problem.
2. Point out what the question is.
3. Identify things you need to know.
4. Decide on a type of strategy you plan to use.
5. Execute your strategy to solve the problem.
6. Remember to check your solution.

Step 1, "Start to read the problem," tells students to read the problem (see Figure 9). This could be done silently or aloud, depending on students' preferences. The first step was designed to help students understand the problem. To facilitate their understanding of the problem, they were allowed to click on the words they did not understand. When clicked, brief definitions of the words were presented along with graphics which visually depicted the concepts. Students also could enlarge the text in the main problem in order to facilitate their reading of the problem. Step 2, "Point out what the question is," tells students what the problem is asking them to find (see Figure 10). In this step, students looked for and highlighted the sentence which asked something and ended with a question mark. Step 3, "Identify things you need to know," prompts students to look for key information to solve the problem (see Figure 11). This is done by asking them to respond to specific questions which contain the relevant information needed to solve the problem. Step 4, "Decide on a type of strategy you plan to use," tells students to decide what kind of strategy they are going to use to solve the

problem based on the strategies taught in the lesson or from their own experience (see Figure 12). Step 5, "Execute the strategy to solve the problem," tells students to carry out the strategy (see Figure 13). Based on the information they obtained from steps one to four, students are prompted with several questions which lead them to execute the strategy step by step. Finally, step 6, "Remember to check the solution," makes students aware of the mathematics problem-solving processes (see Figure 14). This step was designed to help students verify their results by reviewing how they reached their solution. Figures 9 to 14 show screens explaining each step of the SPIDER to solve the problem.

Specific strategy. In the specific strategy session, students were given a description of a specific strategy. They were taught one specific strategy (e.g., Make a table, Work Backwards, or Use logical reasoning) along with the general strategy, "SPIDER", in each lesson.

Practice. In the practice session, students were given opportunities to solve problems using the general strategy, "SPIDER", with one specific strategy. Students were given three major mathematics word problems to practise followed by an optional problem. Each mathematics word problem consisted of approximately 20 sub-questions which were in the format of

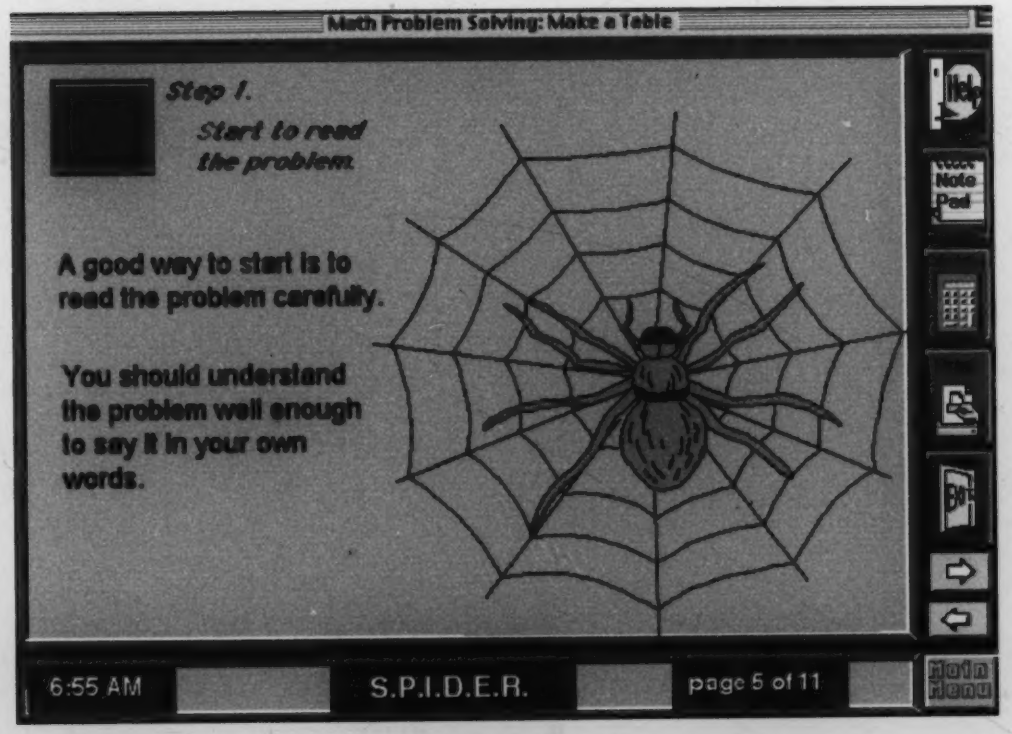

Math Problem Solving: Make a Table
Step 1.
Start to read the problem
A good way to start is to read the problem carefully.
You should understand the problem well enough to say it in your own words.
Help
Note Pad
6:55 AM
S.P.I.D.E.R.
page 5 of 11
Main Menu

Figure 9. Screen explaining first step of SPIDER.

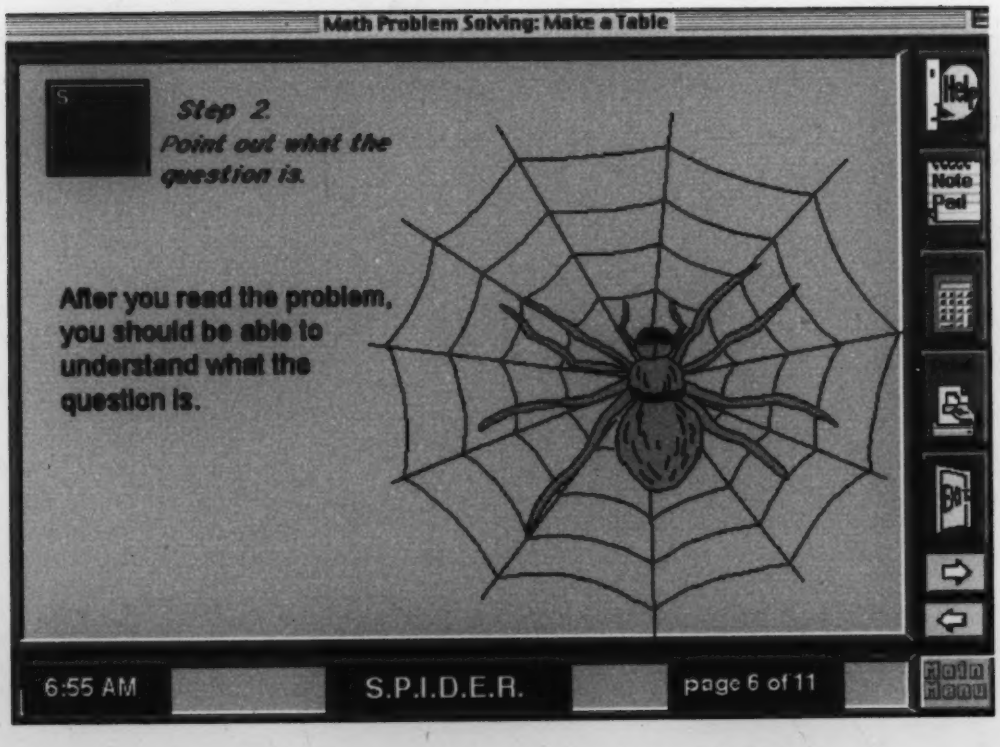



Figure 10. Screen explaining second step of SPIDER.

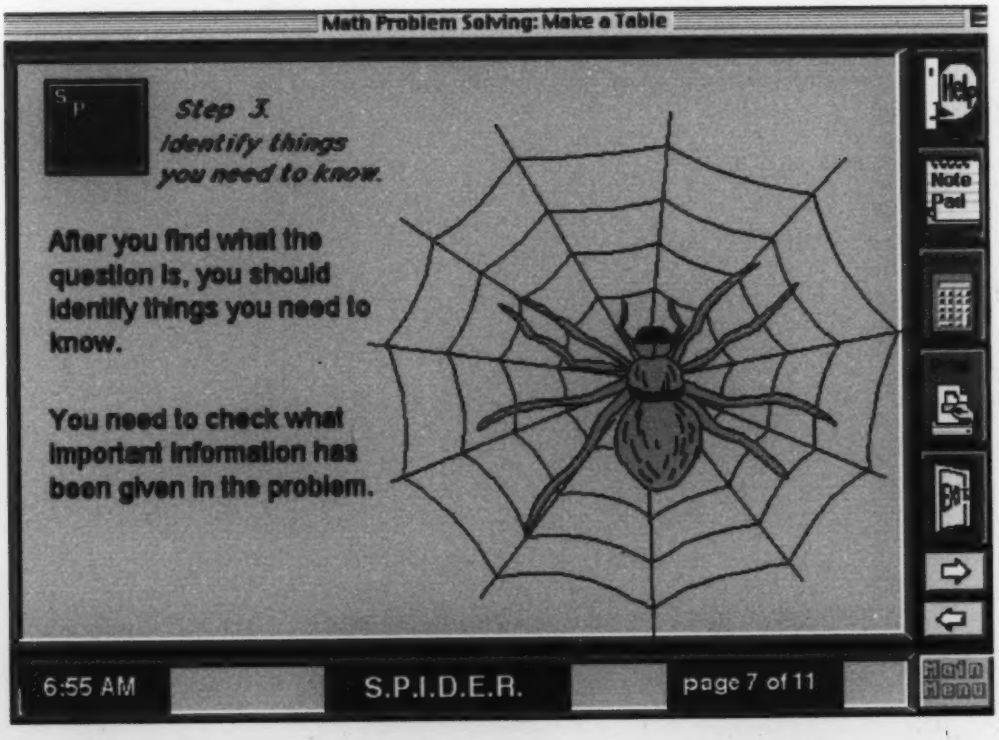

Math Problem Solving: Make a Table
Step 3
Identify things you need to know.
After you find what the question is, you should identify things you need to know.
You need to check what important information has been given in the problem.
Help
Note Pad
6:55 AM
S.P.I.D.E.R.
page 7 of 11
Main Menu

Figure 11. Screen explaining third step of SPIDER.

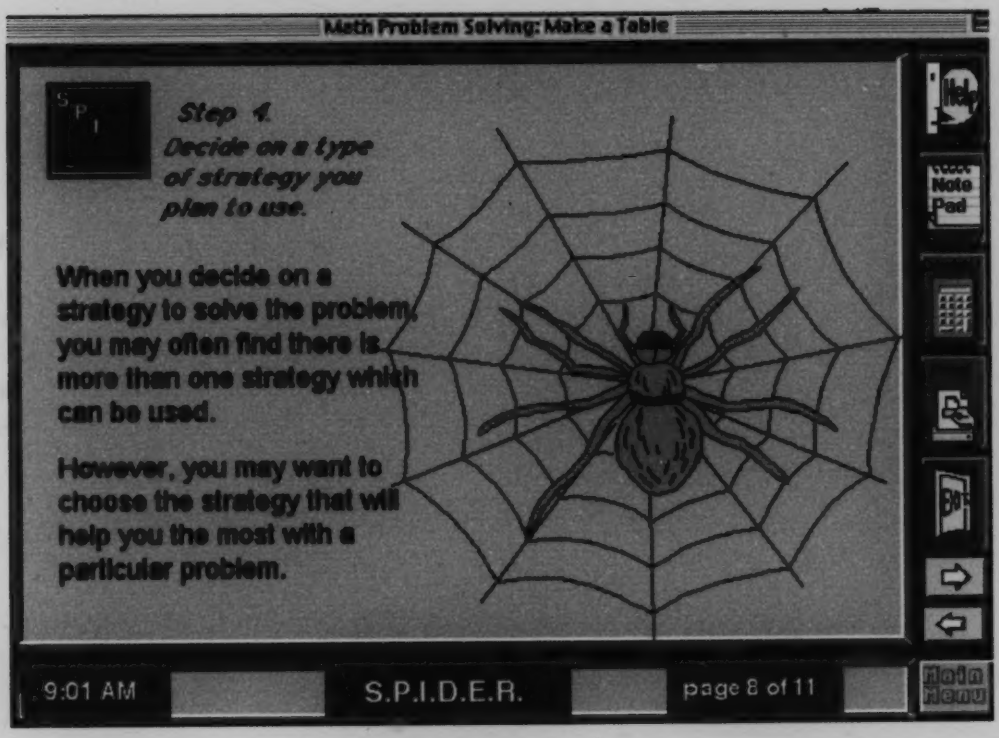

Math Problem Solving: Make a Table
Step 4.
Decide on a type of strategy you plan to use.
When you decide on a strategy to solve the problem, you may often find there is more than one strategy which can be used.
However, you may want to choose the strategy that will help you the most with a particular problem.
Note Pad
9:01 AM
S.P.I.D.E.R.
page 8 of 11
Main Menu

Figure 12. Screen explaining fourth step of SPIDER.

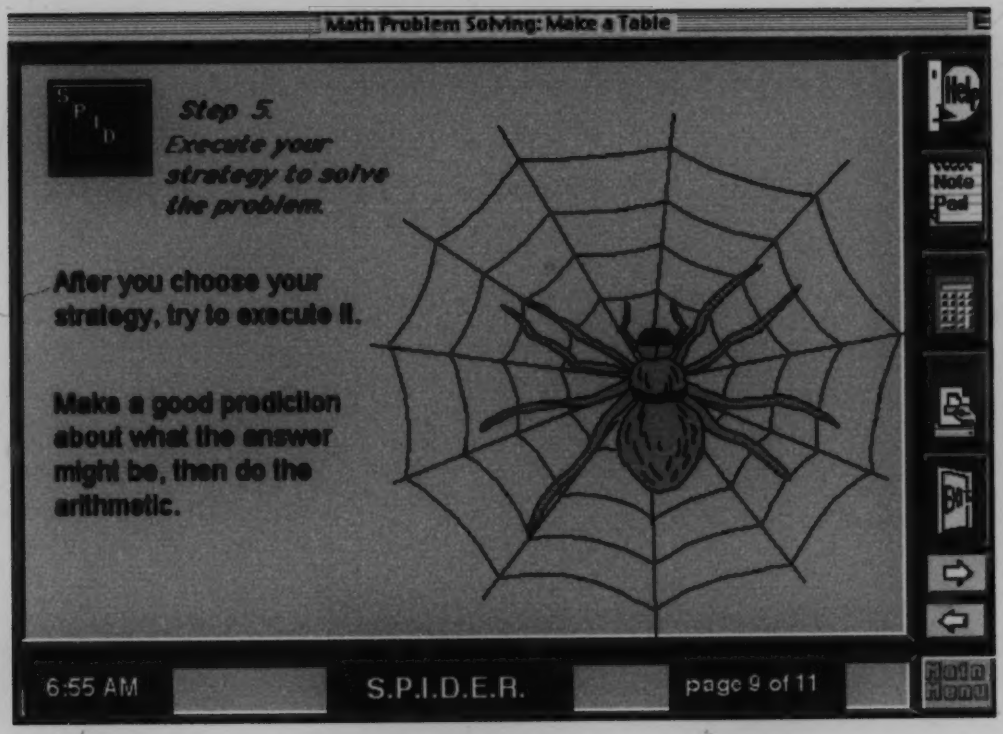

Math Problem Solving: Make a Table
Step 5.
Execute your strategy to solve the problem.
After you choose your strategy, try to execute it.
Make a good prediction about what the answer might be, then do the arithmetic.
Help
Note Pad
6:55 AM
S.P.I.D.E.R.
page 9 of 11
Main Menu

Figure 13. Screen explaining fifth step of SPIDER.

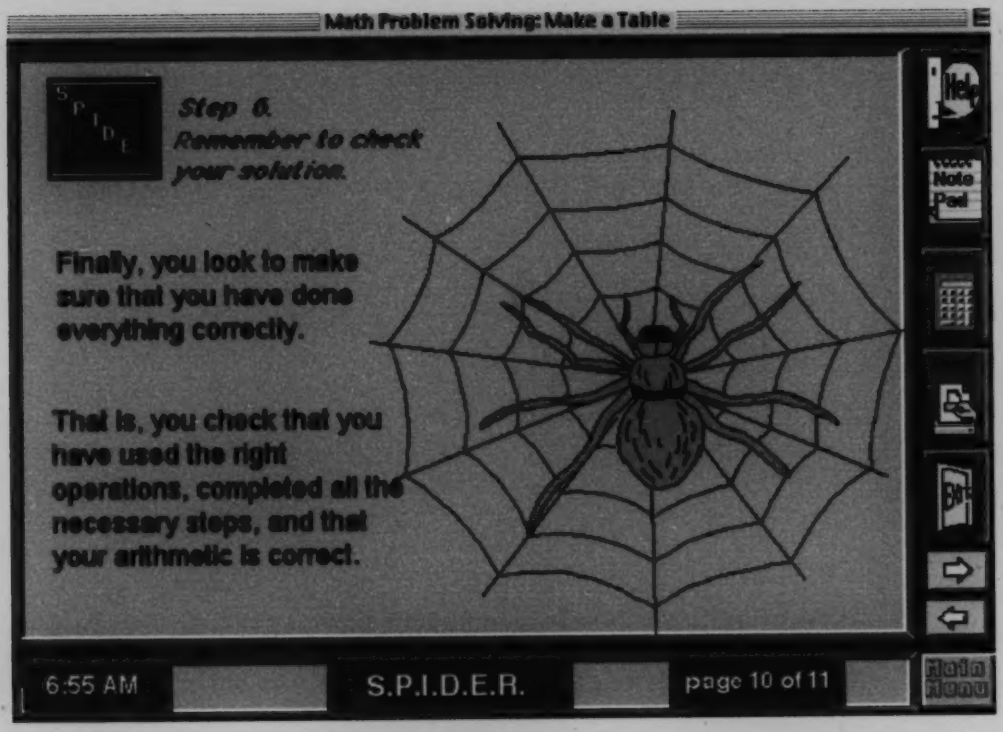

Math Problem Solving: Make a Table
Step 6.
Remember to check your solution.
Finally, you look to make sure that you have done everything correctly.
That is, you check that you have used the right operations, completed all the necessary steps, and that your arithmetic is correct.
Help
Note Pad
6:55 AM
S.P.I.D.E.R.
page 10 of 11
Main Menu

Figure 14. Screen explaining sixth step of SPIDER.

multiple choice, short-answer, marking questions, and dragging screen objects. With regard to the response error, students were given immediate feedback, and also two or three more chances to answer the question. When the students' responses were correct, they were given a variety of encouraging words with animation or an interesting picture.

Quiz. In the quiz session, students were given one major mathematics word problem consisting of approximately 20 sub-questions. Students were not allowed to go back to the previous sub-questions once they had answered and pressed the return key on the keyboard or the right-pointed arrow on the screen. As well, they were not given any feedback regarding the accuracy of the responses during the quiz.

Result. The results of the quiz provided students with information about how they performed on the quiz. Students' answers were evaluated based on the SPIDER model, giving their number of correct answers for each step as well as their self-rated confidence level.

Control-CBI Program (Control-CBI)

Students in the CBI control group received the Control-CBI program which also consisted of three lessons as with the experimental group. However, unlike the PSS-CBI program, each lesson in the Control-CBI

program consisted of only three sections: (1) practice, (2) assessing skills and strategies, and (3) results.

During the practice session, the CBI control group was given the same mathematics word problem as the experimental group. However, the Control-CBI program did not include any type of explicit strategy instruction. In order to ensure that the CBI control and traditional control groups would not have access to the explicit strategies instruction which was given to the experimental group, separate school locations were chosen for all three groups.

The flowcharts for the PSS-CBI (experimental group) and Control-CBI (CBI control group) programs are shown in Figures 3 and 4.

## Instrumentation

Four instruments were used in this study: Mathematics problem solving sub-test from the Canadian Test of Basic Skills (CTBS-MPS), the Mathematics Problem-Solving Test (MPST), the Mathematics Problem-Solving Process Questionnaire (MPSPQ), and an attitude survey.

### Canadian Test of Basic Skills -- Problem-Solving (CTBS-MPS)

The mathematics sub-test (mathematics problem-solving) from the Canadian Test of Basic Skills (multi-level edition, $r = .97$) consisted of 29 multiple choice questions -- 16 single-step and 13 multiple-step problems -- which might be encountered in daily living. This test was administered at

the beginning of the study to measure students' general mathematics problem-solving skills, and also at the end of the study to examine whether the treatment effect had occurred. This test was given on a group basis and took approximately 30 minutes to administer.

Mathematics Problem-Solving Test (MPST)

Pre- and post-MPSTs were developed by the researcher in consultation with experts familiar with teaching and assessing problem-solving skills. During the pre- and post-test phases, all participating students were given three mathematics word problems which were covered in the three lessons: (1) Make a Table, (2) Work backwards, (3) Use Logical Reasoning. Unlike the problems in the CTBS-MPS, these were more complex, process problems which were open-ended in nature with several possible solutions. The purpose of the pre- and post-MPSTs was to measure the effect of explicit strategies instruction. Alternate forms of the MPST were used before and after the treatment.

Each question in the pre- and post- MPSTs consisted of six sub-questions as follows: (1) predict whether you can get the answer correct, (2) point out what the question is, (3) write down all the information you think you need to solve the problem, (4) decide a type of strategy you plan to use, (5) solve the problem showing all of your work, and (6) evaluate how reasonable your answer was. The first and last questions were derived from Flavell (1985) and Slife et al. (1985) to measure knowledge about cognition and

regulation of cognition. The remaining questions were mainly based on the notions of Polya (1957) on mathematics problem-solving processes.

To measure how much progress students made in solution procedures, their responses on pre- and post-MPSTs were scored according to the criteria shown in Table 3. The maximum score for the MPST was 6.

Table 3

Scoring Scales for Pre- and Post- MPSTs

| Number of points | Characteristics of students' solution procedures |
|---|---|
| 0 | • no answer or absolutely wrong answer based on inappropriate approach<br>- showed no understanding of the problem |
| .5 | • 1 answer correct for a problem with multiple answers |
| 1 | • 2 answers correct for a problem with multiple answers<br>• partially correct answer based on appropriate strategy but showed some mistakes. |
| 1.5 | • 3 answers correct for a problem with multiple answers<br>• almost correct answer based on appropriate strategy but showed a minor mistake (e.g., copying error, or computational error) |
| 2 | • 4 or 5 answers correct for a problem with multiple answers<br>• correct answer based on appropriate strategy |

Metacognitive Measures

As metacognitive measures, students' accuracy of self-prediction of performance (ASPP) (i.e., knowledge about cognition) and accuracy of self-evaluation of performance (ASEP) (i.e., regulation of cognition) were examined in each of three problems during the pre- and post-MPSTs.

For the self-prediction of performance measure (SPP), students were asked to read the problem and respond to the item "How sure are you that you will get the answer correct?" Responses were rated on a 4-point Likert-type scale ranging from 1 being "not sure" to 4 being "very sure." With respect to the self-evaluation of performance measure (SEP), students were asked to respond to the item "How reasonable do you think your solution was?" Responses were made after their solution and also scored on a 4-point Likert-type scale, ranging from 1 being "not reasonable" to 4 being "highly reasonable." Each measure consisted of three items for pre- and post-tests respectively.

In order to define ASPP and ASEP, students' mathematics problem scores (i. e., 0, 0.5, 1.0, 1.5, or 2.0) on each major problem in the MPST were compared with their responses of SPP and SEP. A measure of consistency for these sets of ordered states was developed through personal communication with Professor J. R. McGregor, Department of Mathematics Sciences, University of Alberta.

Suppose that two properties each consist of a number of ordered states. Denote by R the property with the greater number (r) of states and denote by C

the property with the fewer number (c) of states. If both properties have the same number of states (r=c) either property can be denoted by R and the other by C. Denote the **r** states of property R by $R_1, R_2, ..., R_n$ and the **c** states of property C by $C_1, C_2, ..., C_c$. By "ordered", we mean that for each i = 1, 2, ..., r-1, $R_i$ has a lower level of property R than does $R_{i+1}$, and that for j = 1, 2, ..., c-1, $C_j$ has a lower level of property C than does $C_{j+1}$.

Let's consider an array of **r** rows and **c** columns of cells such that row i (i = 1, 2, ..., r) is associated with state $\mathbf{R_i}$ and column j (j = 1, 2, ..., c) is associated with $\mathbf{C_j}$. We wish to define a measure of consistency of the states $R_i$ and $C_j$ (i = 1, 2, ..., r; j=1, 2,..., c). As a first step, we define rank ratios r(i, j) for those cell combinations ($R_i$, $C_j$) with subscripts satisfying

$$c(i-1)+j > k$$

where

$$k=cr/2 \text{ if either or both r and c are even}$$

and

$$k=[cr/2] \text{ if c and r are both odd.}$$

(The square brackets denote the integer part of the number contained between them. For example, if c=3 and r=5, [cr/2] = [15/2] = [7.5]=7). For these cell combinations, the rank ratio r(i, j) is the smaller of ic/jr and jr/ic. For the remaining cell combinations, (i.e. those cells for which $c(i-1)+j \leq k$) the rank ratios are defined by the centrosymmetric condition

$$\mathbf{r(i, j) = r(r-i+1, c-j+1)}$$

Having defined all of the rank ratios in this way, we define the consistency x(i, j) of the state $R_i$ and the state $C_j$ to be

$$x(i, j) = (cr(i, j)-1)/(c-1)$$

Thus, for example,

$$x(1, 1) = (cr(1,1)-1)/(c-1) = (c-1)/(c-1) = 1 \quad \textbf{(Upper Left)}$$

$$x(1, c) = (cr(1,c)-1)/(c-1) = (1-1)/(c-1) = 0 \quad \textbf{(Upper Right)}$$

$$x(r, 1) = (cr(r,1)-1)/(c-1) = (1-1)/(c-1) = 0 \quad \textbf{(Lower Left)}$$

$$x(r, c) = (cr(r,c)-1)/(c-1) = (c-1)/(c-1) = 1 \quad \textbf{(Lower Right)}$$

Now, consider that **r** rows of cells refer to each major mathematics problem score in the MPST which range from 0, 0.5, 1.0, 1.5, to 2.0, whereas **c** columns of cells refer to self-prediction and self-evaluation of performance which range from 1 to 4 on a Likert scale: 1 = not sure/not reasonable; 2 = somewhat sure/somewhat reasonable; 3 = sure/reasonable; and 4 = very sure/highly reasonable.

For example,

Those cells for which

$$c(i-1)+j = 4(i-1)+j>10$$

together with their rank ratios and consistencies can be summarized as shown in Table 4 below:

Table 4

Consistency Matrix for $4(i-1)+j \geq 10$

| | Column 1 | Column 2 | Column 3 | Column 4 |
|---|---|---|---|---|
| Row 1 | | | | |
| Row 2 | | | | |
| Row 3 | | | 11*<br>4/5**<br>11/15*** | 12<br>3/5<br>7/15 |
| Row 4 | 13<br>5/16<br>1/12 | 14<br>5/8<br>1/2 | 15<br>15/16<br>11/12 | 16<br>4/6<br>11/15 |
| Row 5 | 17<br>1/4<br>0 | 18<br>1/2<br>1/3 | 19<br>3/4<br>2/3 | 20<br>1<br>1 |

Note. * $4(i-1)+j \geq 10$. ** r (i, j). *** x (i, j).

Now applying the centrosymmetric condition, the relevant results for the remaining cells (those for which $4(i-1)+j \leq 10$) are shown in Table 5 below:

Table 5

Consistency Matrix for $4(i-1)+j \leq 10$

| | Column 1 | Column 2 | Column 3 | Column 4 |
|---|---|---|---|---|
| Row 1 | 1*<br>1** | 2<br>2/3 | 3<br>1/3 | 4<br>0 |
| Row 2 | 5<br>11/15 | 6<br>11/12 | 7<br>1/2 | 8<br>1/12 |
| Row 3 | 9<br>7/15 | 10<br>11/15 | | |
| Row 4 | | | | |
| Row 5 | | | | |

Note. * $4(i-1)+j \leq 10$. ** x (i, j).

Finally, converting to decimals the consistency matrix is shown in Table 6 below:

Table 6

Conversion of the Consistency Matrix into Decimals

| | Column 1 | Column 2 | Column 3 | Column 4 |
|---|---|---|---|---|
| Row 1 | 1 | .67 | .33 | 0 |
| Row 2 | .73 | .92 | .50 | .08 |
| Row 3 | .47 | .73 | .73 | .47 |
| Row 4 | .08 | .50 | .92 | .73 |
| Row 5 | 0 | .33 | .67 | 1 |

In order to measure pre- and post-ASPPs, the ASPPs from each of the three mathematics problems on the MPST were added and then divided by 3 (i.e., the number of questions in the pre- and post- MPSTs). For pre- and post-ASEPs, the same procedure was also carried out using the ASEPs from each of three mathematics problems on the MPST.

Mathematics Problem-Solving Process Questionnaire (MPSPQ)

This questionnaire consisted of a 5-point, Likert-type format of 24 items mainly drawn from the Problem-Solving Questionnaire (Mulcahy, 1987) as a general measure of students' perceptions of problem-solving strategies (see Appendix B). The Likert-scale ranged from "describes me very well",

"describes me well", "describes me somewhat", "does not really describe me" to "does not describe me at all." The 24 items are classified into four groups. The first three groups are components of the cognitive process involved in mathematics problem-solving (i.e., orientation, organization, execution). The fourth group is a component of the metacognitive process involved in mathematics problem-solving (i.e., verification) as suggested by Flavell (1985) and Lester (1985). With regard to the measure of internal consistency, Cronbach's alpha for the MPSPQ was estimated to be .80, indicating a high degree of reliability. The maximum score for the MPSPQ was 120.

Attitude Survey

The attitude survey contained 12 items to which students responded using the same 5-point Likert-scale used with the MPSPQ. The survey was designed to assess attitudes toward mathematics, mathematics problem-solving, computer use, and the CBI program. Below the 12 items was an open-ended section in which the students were encouraged to add any additional comments concerning the CBI program. This attitude survey was given to students in the experimental and CBI control groups after they had completed the CBI program. Students in the traditional control group did not receive any type of CBI program, and therefore were asked to complete only the first 3 items of the attitude survey during the post-test phase. These three items were on attitudes toward mathematics, mathematics problem-solving, and computer use. Tables 18 and 19 present a list of the items included on

this survey. Each item was scored by assigning values to each response and subsequently analyzed individually using the Chi-square test to ascertain if there was a significant difference between the experimental and CBI control groups. Values of 1 to "describes me very well" or "describes me well", and 0 to "does not really describe me" or "does not describe me at all", were assigned. Responses of "describes me somewhat" were not included in the Chi-Square analyses to avoid ambiguity.

## Procedure

This study was divided into three stages: (1) pre-tests (three weeks), (2) intervention (three weeks), and (3) post-tests (two weeks).

During the first stage, the students were given pre-tests of a Mathematics Problem-Solving sub-test from the Canadian Test of Basic Skills (CTBS-MPS), Mathematics Problem-solving Test (MPST), and the Mathematics Problem-Solving Process Questionnaire (MPSPQ).

As described earlier, during the second stage of this study, students in the experimental group were exposed to the CBI program with explicit strategy instruction, whereas students in the CBI control group were exposed to the CBI program without explicit strategy instruction. All the students in the experimental and CBI control groups used Apple Macintosh computers in the computer laboratory at their school. Basic instruction was provided on the use of the CBI program at the beginning of the second stage of this study. For the experimental and CBI control groups, CBI lessons were conducted

during the students' computer lab time for a 60-70 minute period per week over a duration of three weeks. The researcher and the teacher were available to deal with technical difficulties that occurred with the computer, and to provide individual assistance upon request. For those students who were not able to finish the lesson, they were allowed to complete the lesson during lunch hours or after school. The researcher was also present to answer questions during that time. Students were given as much time as needed to complete the lesson. Students in the traditional control group did not receive any type of CBI program during this study; furthermore, they were not informed that they were in this group. In order to give all students equal benefit from participating, the CBI program was provided for this group after the study was concluded.

During the third stage, all the students were given post-tests using the CTBS-MPS, MPST, and MPSPQ. They were also given an attitude survey on the CBI program, mathematics, and mathematics problem-solving.

## Data Analysis

To measure the effectiveness of explicit mathematics problem-solving strategy instruction using the CBI program, data were analyzed using 3 x 2 x 2 (Group [experimental, CBI control, and traditional control] x Mathematics Problem-Solving Achievement Group [high and low] x Time of Testing [pre- and post-tests]) Analysis of Variance with repeated measures on the last factor (time of testing).

To examine students' improvement in their mathematics problem-solving performance, the pre- and post- CTBS-MPSs and MPSTs were used. To investigate students' improvement of their metacognitive skills in mathematics problem-solving, accuracy of self-prediction of performance (ASPP) as knowledge about cognition, and accuracy of self-evaluation of performance (ASEP) as regulation of cognition components were examined. Self-prediction of performance (SPP) and self-evaluation of performance (SEP) were inferred by totaling the results from each question in pre- and post- Mathematics Problem-Solving Tests (MPSTs) and dividing by 3 (i.e., the number of questions in the pre- and post- MPSTs). With regard to the ASPP and ASEP measures, the decimal conversion of the consistency matrix was used, as shown in Table 6, based on the results of pre- and post- measures of SEP, SPP, and MPST. To examine students' attitudes toward mathematics, mathematics problem-solving, and the CBI programs (i.e., PSS-CBI and Control-CBI programs), data were analyzed using $\chi^2$ tests.

Independent variables for research questions 1 to 6 were three treatment conditions (experimental, CBI control, and traditional control) and two mathematics problem-solving achievement levels (high and low) based on the results of the mathematics problem-solving sub-test from the Canadian Test of Basic Skills (multi-level edition). The independent variable for research question 7 was three treatment conditions (experimental, CBI control, and traditional control), whereas for research question 8 was only two treatment conditions (experimental and CBI control). Dependent variables

were CTBS-MPS, MPST for research question 1, SPP for research question 2, SEP for research question 3, ASPP for research question 4, ASEP for research question 5, MPSPQ for research question 6, and the attitude survey for research questions 7 and 8. Dependent variables, with the exception of the attitude survey, were administered as pre- and post-tests. A summary of this design is shown in Table 7.

Table 7

Research Design

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Experimental | O1 | O2 | O3 | O4 | X1a | X2a | X3a | O5 | O6 | O7 | O8 |
| CBI Control | O1 | O2 | O3 | O4 | X1b | X2b | X3b | O5 | O6 | O7 | O8 |
| Traditional Control | O1 | O2 | O3 | O4 | | | | O5 | O6 | O7 | O8 |

X1a: Treatment 1 (Make a table)

X2a: Treatment 2 (Work backwards)

X3a: Treatment 3 (Use logical reasoning)

X1b: Treatment 4 (Make a table)

X2b: Treatment 5 (Work backwards)

X3b: Treatment 6 (Use logical reasoning)

O1, O5: a sub-test from the Canadian Test of Basic Skills (CTBS-MPS)

O2, O6: Mathematics Problem-Solving Process Test (MPST)

O3, O7: Self-prediction and self-evaluation measures on the MPST

O4, O8: Mathematics Problem-Solving Process Questionnaire (MPSPQ)

# V. Results

This chapter presents the results of the data analyses and contains four parts. Part one, analyses of initial participant characteristics, reports the results of t-tests on each of six pre-test measures with the exception of the attitude measure. Part two, reliability measures on scoring procedures of the Mathematics Problem-Solving Test (MPST), reports the results of interrater reliability. Part three, analyses of the Canadian Test of Basic Skills-Mathematics Problem-Solving (CTBS-MPS), MPST, Self-Prediction of Performance (SPP), Self-Evaluation of Performance (SEP), Accuracy of Self-Prediction of Performance (ASPP), and Accuracy of Self-Evaluation of Performance (ASEP), and Mathematics Problem-Solving Process Questionnaire (MPSPQ), reports the results of a 3 x 2 x 2 (Group [experimental, CBI control, and traditional control] x Mathematics Problem-Solving Achievement Group [high and low] x Time of Testing [pre- and post-tests]) factorial, with repeated measures on the Time factor. These Analyses of Variance with repeated measures provide the answers for research questions 1, 2, 3, 4, 5, and 6. The final part, analyses of the attitude survey, reports the results of Chi-square tests on each item and one open-ended question. Research questions 7 and 8 are examined in the final part.

## Comparison of Student Performance by Mathematics Problem-Solving Achievement Groups (High and Low)

To examine whether there was a significant difference on initial participant characteristics between high and low achieving students, t-tests were conducted on each of the six pre-test measures with the exception of the attitude survey. Table 8 contains a summary of the findings which compared students' results in the high achieving group with those in the low achieving group on the MPST, self-prediction of performance, self-evaluation of performance, accuracy of self-prediction of performance, accuracy of self-evaluation of performance, and the MPSPQ. The findings indicated that students in the high achieving group outperformed those in the low achieving group on the MPST, $t\,(103) = 4.99$, $p < .001$, on the MPSPQ, $t\,(97) = 3.30$, $p < .01$, on self-prediction of performance, $t\,(79) = 3.94$, $p < .001$, and on self-evaluation of performance, $t\,(93) = 3.58$, $p < .01$. That is, students in the high achieving group obtained a higher mean score on the mathematics problem-solving test than those in the low achieving group. As well, high achieving students were more optimistic about their solutions. However, there were no significant differences observed between high and low achieving groups on the metacognitive measures as shown in the results of accuracy of self-prediction of performance, $t\,(74) = .73$, $p > .05$, and accuracy of self-evaluation of performance, $t\,(90) = .08$, $p > .05$.

Reliability Measures on Scoring Procedures of Pre- and Post-MPSTs

As pre- and post- MPSTs were in the format of open-ended questions, they were scored by the researcher (Rater 2) based on the scoring system developed by the researcher (see Table 3). In determining the reliability of scoring procedures, approximately 30 percent of the students' responses on the pre- and post-MPSTs were randomly selected using SPSS and scored individually by a research assistant (Rater 1) who is a graduate student in the

Table 8

Performance Differences of Six Pre-test Measures

| | High | | | Low | | | |
|---|---|---|---|---|---|---|---|
| | n | m | SD | n | m | SD | t |
| MPST | 57 | 3.25 | 1.11 | 48 | 2.17 | 1.10 | 4.99*** |
| Self-Prediction of Performance (SPP) | 43 | 2.95 | .51 | 38 | 2.47 | .59 | 3.94*** |
| Self-Evaluation of Performance (SEP) | 52 | 3.24 | .61 | 43 | 2.83 | .50 | 3.58** |
| Accuracy of Self-Prediction of Performance (ASPP) | 41 | .66 | .13 | 35 | .64 | .15 | .73 |
| Accuracy of Self-Evaluation of Performance (ASEP) | 50 | .63 | .20 | 42 | .63 | .16 | .08 |
| MPSPQ | 55 | 86.87 | 11.41 | 44 | 79.36 | 11.08 | 3.30** |

Note. *p < .05. **p < .01. ***p < .001.

Department of Educational Psychology at the University of Alberta. Rater 1 was instructed by the researcher how to score students' responses according to the scoring system. As students' identification numbers were recoded during the scoring procedures, the rater did not know which one belonged to the experimental, the CBI control, or the traditional control groups, nor which belonged to the pre- or post- MPST. Table 9 shows the means and standard deviations of the pre- and post-MPST scores for randomly selected students (n = 30) from the combined three groups.

Table 9

Means and Standard Deviations of Pre- and Post- MPST Scores

| | Pre-MPST | | | Post-MPST | | |
|---|---|---|---|---|---|---|
| | n | m* | SD | n | m* | SD |
| Rater 1 | 30 | 3.12 | 1.38 | 30 | 3.93 | 1.36 |
| Rater 2 | 30 | 3.20 | 1.34 | 30 | 4.05 | 1.27 |

Note. * Maximum score was 6.

As shown in Table 9, there was some discrepancy between the two raters on the pre- and post-MPST measures. However, the results of paired t-tests showed that there were no significant differences between the raters' judgments on the pre-MPST, $t(29) = -1.72$, $p > .05$, and post-MPST, $t(30) = -1.56$, $p > .05$.

With regard to the interrater reliability, the correlation coefficients between the raters were computed, resulting in alpha coefficient .99, that is, .99 for the pre-MPST measure; .98 for the post-MPST measure. These coefficients indicated that there was very high interrater agreement between the two raters on these measures. Therefore, it was considered that the researcher's (Rater 2) judgment was fairly reliable on the MPST measure.

## Performance by Group by Achievement Group

Research Question 1: Are there significant mean score differences on mathematics problem-solving performance for achievement groups (high and low) among three groups (experimental, CBI control, and traditional control)?

In order to examine research question 1, the CTBS-MPS and MPST measures were administered. The results of these measures are described below.

Performance on CTBS-MPS. Table 10 shows the means and standard deviations for the mathematics problem-solving achievement groups (high and low) within the experimental, CBI control, and traditional control groups. The results of the 3 x 2 x 2 (Group x Mathematics Problem-Solving Achievement x Time of Testing) univariate factorial ANOVA with repeated measures on the last factor showed that there was an interaction effect for Group x Time, $F(99, 2) = 3.61$, $p < .05$. The results suggested that the pre-test -

Table 10

A Summary of Means and Standard Deviations for the Pre- and Post-tests on Dependent Variables

| Variables | | Experimental | | | CBI Control | | | Traditional Control | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | High (n=19) | Low (n=20) | Total (n=39) | High (n=18) | Low (n=11) | Total (n=29) | High (n=20) | Low (n=17) | Total (n=37) |
| CTBS-MPS | Pre- | 20.89 | 10.55 | 15.59 | 21.72 | 10.45 | 17.45 | 22.95 | 10.65 | 17.30 |
| | | (3.02) | (3.27) | (6.09) | (3.37) | (2.84) | (6.38) | (3.62) | (3.08) | (7.06) |
| | Post- | 21.53 | 14.70 | 18.03 | 22.06 | 11.55 | 18.07 | 22.80 | 11.53 | 17.62 |
| | | (4.29) | (3.99) | (5.35) | (3.89) | (4.61) | (6.61) | (3.83) | (3.08) | (6.66) |
| MPST | Pre- | 3.24 | 2.33 | 2.77 | 3.14 | 1.68 | 2.57 | 3.35 | 2.29 | 2.86 |
| | | (1.02) | (1.10) | (1.15) | (1.26) | (.90) | (1.33) | (1.09) | (1.17) | (1.23) |
| | Post- | 4.82 | 3.33 | 4.05 | 4.36 | 3.41 | 4.00 | 4.30 | 3.38 | 3.88 |
| | | (.95) | (1.15) | (1.27) | (.89) | (1.20) | (1.10) | (.78) | (1.34) | (1.16) |
| MPSPQ | Pre- | 85.47 | 80.32 | 82.89 | 86.56 | 77.89 | 83.44 | 88.45 | 79.06 | 84.28 |
| | | (14.29) | (12.45) | (13.47) | (9.08) | (10.51) | (10.32) | (10.34) | (10.23) | (11.19) |
| | Post- | 86.37 | 78.28 | 82.43 | 85.71 | 80.45 | 83.64 | 88.85 | 79.65 | 84.62 |
| | | (14.58) | (13.73) | (14.56) | (13.35) | (11.05) | (12.56) | (8.31) | (11.30) | (10.71) |
| SPP | Pre- | 2.96 | 2.63 | 2.78 | 2.92 | 2.58 | 2.79 | 2.98 | 2.24 | 2.62 |
| | | (.56) | (.63) | (.61) | (.45) | (.56) | (.51) | (.53) | (.51) | (.63) |
| | Post- | 3.48 | 2.96 | 3.22 | 3.37 | 3.09 | 3.26 | 3.15 | 2.90 | 3.04 |
| | | (.57) | (.86) | (.77) | (.60) | (.68) | (.64) | (.44) | (.45) | (.46) |
| SEP | Pre- | 3.11 | 2.87 | 2.99 | 3.40 | 2.96 | 3.24 | 3.25 | 2.71 | 3.00 |
| | | (.82) | (.51) | (.68) | (.42) | (.56) | (.52) | (.49) | (.44) | (.54) |
| | Post- | 3.16 | 3.14 | 3.29 | 3.56 | 3.18 | 3.41 | 3.27 | 2.98 | 3.14 |
| | | (.64) | (.80) | (.74) | (.47) | (.58) | (.54) | (.50) | (.56) | (.54) |

(table continues)

| Variables | | Experimental | | | CBI Control | | | Traditional Control | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | High (n=19) | Low (n=20) | Total (n=39) | High (n=18) | Low (n=11) | Total (n=29) | High (n=20) | Low (n=17) | Total (n=37) |
| Accuracy of SPP | Pre- | .65 (.14) | .67 (.12) | .66 (.13) | .66 (.11) | .59 (.22) | .63 (.16) | .68 (.13) | .65 (.12) | .67 (.12) |
| | Post- | .78 (.12) | .57 (.15) | .67 (.17) | .70 (.17) | .66 (.11) | .69 (.15) | .73 (.15) | .66 (.14) | .70 (.15) |
| Accuracy of SEP | Pre- | .62 (.22) | .62 (.16) | .62 (.19) | .60 (.19) | .60 (.20) | .60 (.19) | .67 (.19) | .65 (.13) | .66 (.16) |
| | Post- | .76 (.16) | .60 (.16) | .67 (.17) | .73 (.18) | .61 (.10) | .69 (.16) | .78 (.13) | .70 (.15) | .74 (.14) |

post-test difference on CTBS-MPS was greater for the experimental group (2.44) than for the CBI control (.62) and the traditional control (.32) groups. As shown in Figure 15, a significant interaction effect indicated that the pre-test - post-test difference for the experimental group was reliably greater than for the other two groups. The interaction between the Math Achievement Group and Time factors was also found to be significant, $F(99, 1) = 6.52$, $p < .05$, indicating that the pre- post-test difference for the low achieving students (2.29) was reliably greater than for the high achieving students (.26) on this measure (see Table 11, Figure 16, and Table A1). No significant interaction effect for Group x Mathematics Achievement Group x Time of testing was observed.

Table 11

Means and Standard Deviations of the Pre- and Post- CTBS-MPS Scores

| | Pre-CTBS-MPS | | Post-CTBS-MPS | |
|---|---|---|---|---|
| | m* | SD | m* | SD |
| High[a] | 21.88 | 3.40 | 22.14 | 3.97 |
| Low[b] | 10.56 | 3.05 | 12.85 | 4.08 |
| Total | 16.70 | 6.52 | 17.90 | 6.13 |

Note. * Maximum score = 29. [a]n = 57 [b]n = 48

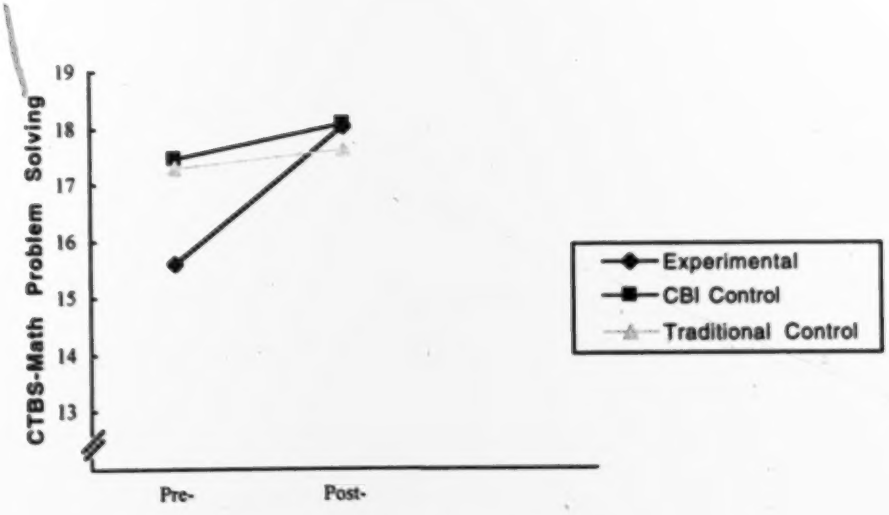



Figure 15. Pre- and Post- CTBS-MPS Performance for Experimental, CBI Control, and Traditional Control Groups.

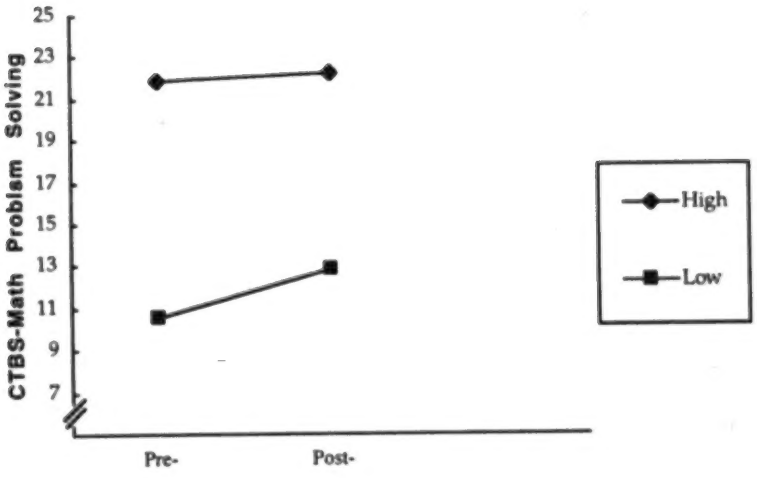



Figure 16. Pre- and Post- CTBS-MPS Performance for High and Low Achievement Groups.

Mathematics Problem-Solving Tests (MPST). Table 10 displays the means and standard deviations for the pre- and post-MPST measures among the three groups. Total mathematics problem-solving scores were subjected to the 3 x 2 x 2 (Group x Mathematics Problem-Solving Achievement Group x Time of Testing) Analysis of Variance with the repeated measures on the last factor. Table A2 shows that there was a main effect for the mathematics problem-solving achievement group, $F(98,1) = 75.43$, $p < .001$, indicating the high achieving students outperformed the low achieving students on this measure (see Figure 17 and Table 12). However, there were no significant interaction effects observed.

Table 12

Means and Standard Deviations of the Pre- and Post- MPST Scores

| | Pre-MPST | | Post-MPST | |
|---|---|---|---|---|
| | m* | SD | m* | SD |
| High[a] | 3.25 | 1.11 | 4.49 | .89 |
| Low[b] | 2.17 | 1.10 | 3.36 | 1.21 |
| Total | 2.75 | 1.22 | 3.98 | 1.18 |

Note. *Maximum score = 6. [a]n = 57 [b]n = 48

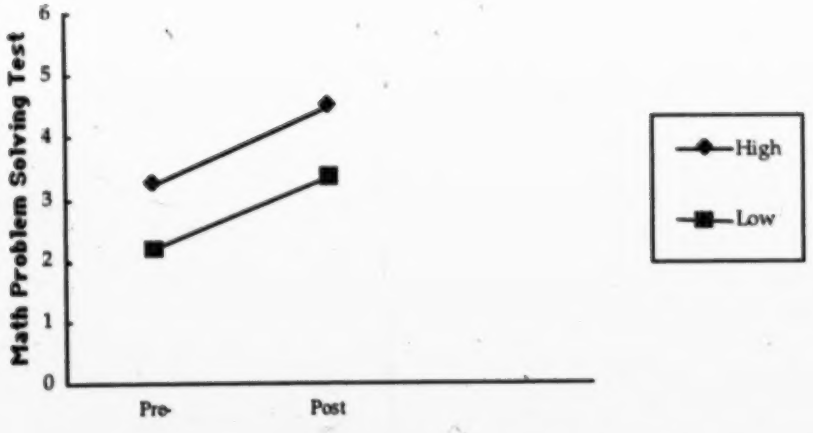



Figure 17 Pre- and Post-MPST Performance for High and Low Achievement Groups.

Research Question 2: Are there significant mean score differences on SPP for achievement groups (high and low) among the three groups (experimental, CBI control, and traditional control)?

The results of the 3 x 2 x 2 (Group x Mathematics Problem-Solving Achievement Group x Time of Testing) Analysis of Variance with repeated measures on the Time factor showed that there was a main effect for mathematics problem-solving achievement group, $F(73,1) = 13.32$, $p < .001$, indicating that the high achieving students provided higher self-prediction of performance than the low achieving students (see Figure 18, Table 13 , and Table A3). However, there were no significant interaction effects observed.

Table 13

Means and Standard Deviations of the Pre- and Post- Self-Prediction of Performance (SPP) Scores

| | Pre-SPP | | Post-SPP | |
|---|---|---|---|---|
| | m* | SD | m* | SD |
| High[a] | 2.95 | .51 | 3.33 | .55 |
| Low[b] | 2.47 | .59 | 2.97 | .68 |
| Total | 2.73 | .59 | 3.17 | .64 |

Note. *Maximum score = 4. [a]n = 57 [b]n = 48

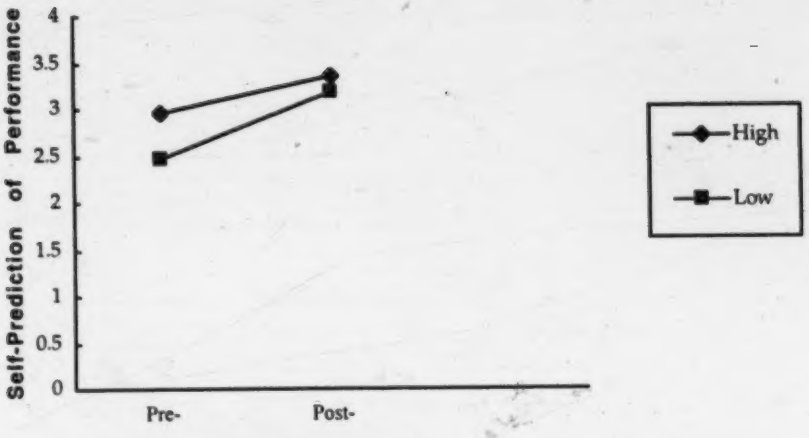



Figure 18. Pre- and Post-SPP Results for High and Low Achievement Groups.

Research Question 3: Are there significant mean score differences on SEP for achievement groups (high and low) among the three groups (experimental, CBI control, and traditional control)?

The results of the three-way ANOVA with repeated measures on the Time factor show that there was a main effect for mathematics problem-solving achievement group, $F(83,1) = 11.54$, $p < .01$, indicating that the high achieving students rated their performance as higher than did the low achieving students (see Figure 19, Table 14, and Table A4). However, significant interaction effects were not observed.

Table 14

Means and Standard Deviations of the Pre- and Post- Self-Evaluation of Performance (SEP) Scores

| | Pre-SEP | | Post-SEP | |
|---|---|---|---|---|
| | m* | SD | m* | SD |
| High[a] | 3.24 | .61 | 3.42 | .54 |
| Low[b] | 2.83 | .50 | 3.10 | .67 |
| Total | 3.06 | .59 | 3.27 | .62 |

Note. *Maximum score = 4. [a]n = 57 [b]n = 48

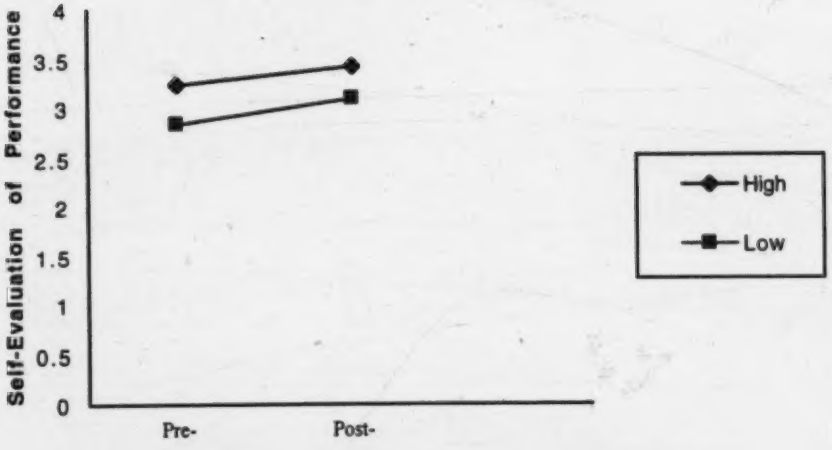



Figure 19. Pre- and Post-SEP Results for High and Low Achievement Groups.

Research Question 4: Are there significant mean score differences on knowledge about cognition (i.e., ASPP) for achievement groups (high and low) among the three groups (experimental, CBI control, and traditional control)?

The results of the 3 x 2 x 2 (Group x Mathematics Problem-Solving Achievement Group x Time of Testing) Analysis of Variance with repeated measures on the Time factor show that there was a main effect for mathematics problem-solving achievement group, $F(68,1) = 7.27$, $p < .01$, indicating that the high achieving students were more accurate in their self-prediction of performance than were the low achieving students (see Tables 15 and A5). However, significant interaction effects were not observed.

Table 15

Means and Standard Deviations of the Pre- and Post- Accuracy of Self-Prediction of Performance (ASPP) Scores

| | Pre-ASPP | | Post-ASPP | |
|---|---|---|---|---|
| | m | SD | m | SD |
| High[a] | .66 | .13 | .74 | .15 |
| Low[b] | .64 | .15 | .63 | .14 |
| Total | .65 | .14 | .69 | .16 |

Note. [a]n = 57 [b]n = 48

Research Question 5: Are there significant mean score differences on regulation of cognition (i.e., ASEP) for achievement groups (high and low) among the three groups (experimental, CBI control, and traditional control)?

The results of the three-way ANOVA with repeated measures on the Time factor show that there was a main effect for mathematics problem-solving achievement group, $F(79,1) = 5.75$, $p < .05$, indicating that the high achieving students were more accurate in their self-evaluation of performance than were the low achieving students (see Tables 16 and A6). However, significant interaction effects were not observed.

Table 16

Means and Standard Deviations of Pre- and Post- Accuracy of Self-Evaluation of Performance (ASEP) Scores

| | Pre-ASEP | | Post-ASEP | |
|---|---|---|---|---|
| | m | SD | m | SD |
| High[a] | .63 | .20 | .76 | .15 |
| Low[b] | .63 | .16 | .64 | .15 |
| Total | .63 | .18 | .70 | .16 |

Note. [a]n = 57 [b]n = 48

Research Question 6: Are there significant mean score differences on the perception of mathematics problem-solving processes for achievement groups (high and low) among the three groups (experimental, CBI control, and traditional control)?

The total scores obtained on the MPSPQ were subjected to the 3 x 2 x 2 (Group x Mathematics Problem-Solving Achievement Group x Time of Testing) Analysis of Variance with repeated measures on the last factor. A significant main effect was observed for the Mathematics Problem-Solving Achievement Group, indicating that the high achieving students outperformed the low achieving students on this measure, $F$ (91, 1) = 10.63, $p$ < .01 (see Figure 20, Table 17, and Table A7). However, there were no interaction effects for Group x Time, Mathematics Problem-Solving Achievement Group x Time, and Group x Mathematics Problem-Solving Achievement x Time (see Table A7).

Table 17

Means and Standard Deviations of the Pre- and Post- MPSPQ Scores

| | Pre-MPSPQ | | Post-MPSPQ | |
|---|---|---|---|---|
| | m | SD | m | SD |
| High[a] | 86.87 | 11.41 | 87.05 | 12.13 |
| Low[b] | 79.36 | 11.08 | 79.30 | 12.02 |
| Total | 83.54 | 11.82 | 83.56 | 12.63 |

Note. [a]n = 57 [b]n = 48

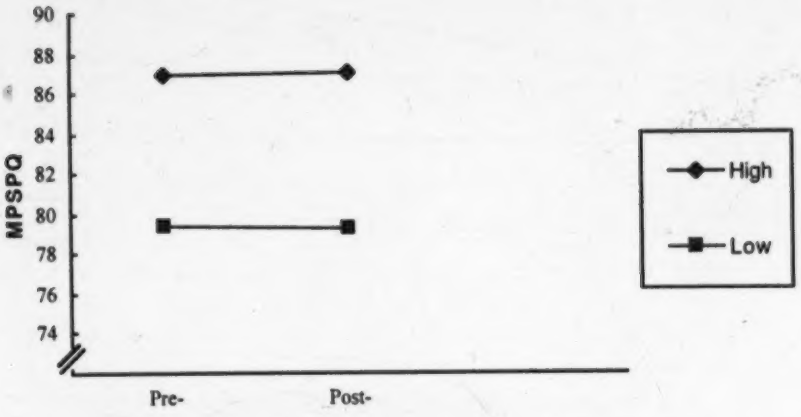



Figure 20. Pre- and Post- MPSPQ Results for High and Low Achievement Groups.

## Summary

The findings of this study revealed that students in the experimental group showed significantly higher mean gain scores (pre-test--post-test mean differences) than those in the CBI control and traditional control groups on the CTBS-MPS measure. A majority of students who were exposed to the explicit problem-solving strategies embedded CBI program (experimental group) achieved higher mean gain scores than both those exposed to the Control-CBI program (CBI control group) and those not exposed to any type of CBI program.

With respect to the MPST, MPSPQ, SPP, SEP, ASPP, and ASEP measures, there were significant differences observed between the high and low achieving groups, indicating that the high achieving group surpassed the low achieving group. However, there were no significant differences

observed among the three groups, although the experimental group did obtain higher mean gain scores on self-evaluation of performance (experimental = .30, CBI control = .17, and traditional control = .14).

## Attitudes

Of the 105 students in this study, 68 students in the experimental and CBI control groups completed the entire survey, consisting of 12 Likert-scale items and an open-ended question, whereas 37 students in the traditional control group responded to only the first three items in the survey. With regard to the measure of internal consistency, Cronbach's alpha for the 12 Likert-scale items was estimated to be .85, indicating a high degree of reliability. The results of the attitude survey are described below.

Research Question 7: Are there significant differences on attitudes toward mathematics, mathematics problem-solving, and computer use among the three groups (experimental, CBI control, and traditional control)?

A comparison was made among the three groups' (experimental, CBI control, and traditional control) responses with respect to their attitudes toward mathematics, mathematics problem-solving, and computer use. Using the Chi-square tests, statistical significance levels were found, as shown in Table 18, on attitudes toward mathematics, $\chi^2$ (2, $N$ = 83) = 16.81, $p < .001$, and mathematics problem-solving, $\chi^2$ (2, $N$ = 75) = 12.69, $p < .001$, among the

three groups. The findings showed that a significant number of students in the experimental and CBI control groups reported that they favored mathematics (experimental = 74.2% and CBI control = 89.3%) and mathematics problem-solving (experimental = 79.3% and CBI control = 80.0%), whereas only 37.5% of the students in the traditional control group responded that they liked math, and 38.5 % stated that they favored mathematics problem-solving (items #1 and 2).

Table 18

A Summary of Chi-Square Analyses for the Three Groups on Attitudes toward Mathematics, Mathematics Problem-Solving, and Computer Use.

| | | Describes me | | | Does not describe me | | | |
|---|---|---|---|---|---|---|---|---|
| | Items | Exp | CBI Ctr. | Trd Ctr. | Exp | CBI Ctr. | Trd Ctr. | $\chi^2$ |
| 1 | I like math. | 74.2% *23/31** | 89.3% *25/28* | 37.5% *9/24* | 25.8% *8/31* | 10.7% *3/28* | 62.5% *15/24* | 16.81*** |
| 2 | I like mathematics problem-solving. | 79.3% *23/29* | 80.0% *16/20* | 38.5% *10/26* | 20.7% *6/29* | 20.0% *4/20* | 61.5% *16/26* | 12.69** |
| 3 | I am interested in using the computer. | 94.7% *36/38* | 100.0% *28/28* | 100.0% *36/36* | 5.3% *2/38* | 0% *0/28* | 0% *0/36* | 3.44 |

Note. * Number of respondents/total sample size in each group.

Total sample size for each group varies because some of the students gave a "describes me somewhat" response so these data were not used in the Chi-square analyses.

**$p < .01$. ***$p < .001$.

With respect to their attitudes toward computer use, however, there was no significant difference observed among the three groups, $\chi^2$ (2, $\underline{N}$ = 102) = 3.44, $\underline{p} > .05$. Regardless of the group, an overwhelming majority of students responded that they were interested in using the computer (experimental = 94.7%, CBI control = 100%, and traditional control = 100%).

Research Question 8: Is there a significant difference between the experimental and CBI control groups on their attitudes toward the CBI program which they received?

To measure the effects of the explicit strategies embedded CBI program on attitudes, students in the experimental and the CBI control groups were given nine Likert-scale questions on attitudes about mathematics problem-solving, and as well as the CBI program they received. They were also asked for additional comments on the CBI program. Overall, responses of the students in the experimental and CBI control groups were overwhelmingly positive on the questions toward computer use, regardless of whether they received the CBI program with explicit strategies instruction. It was observed that there was no significant difference on students' attitudes toward the CBI program between the experimental and CBI control groups. The results are summarized in Table 19.

Table 19
A Summary of Chi-Square Analysis for the Experimental and CBI Control Groups on Attitudes toward Mathematics Problem-Solving and the CBI Program.

| | Items | Describes me | | Does not describe me | | $\chi^2$ |
|---|---|---|---|---|---|---|
| | | Exp. | CBI Ctr. | Exp. | CBI Ctr. | |
| 4 | I didn't like mathematics problem-solving before I tried the CBI program. | 63.3% *19/30** | 75% *18/24* | 36.7% *11/30* | 25% *6/24* | .84 |
| 5 | It was fun to try the CBI program. | 94.7% *6/38* | 100% *29/29* | 5.3% *2/38* | 0% *0/29* | 1.57 |
| 6 | I felt that the computer was my private tutor. | 73.5% *25/34* | 82.6% *19/23* | 26.5% *9/34* | 17.4% *4/23* | .64 |
| 7 | Taking the CBI program made it possible for me to work at my own pace. | 88.9% *32/36* | 92.6% *25/27* | 11.1% *4/36* | 7.4% *2/27* | .25 |
| 8 | The CBI program made me more interested in mathematics problem-solving. | 80.6% *25/31* | 92.6% *25/27* | 19.4% *6/31* | 7.4% *2/27* | 1.73 |
| 9 | I like the CBI program better than classroom instruction. | 89.2% *33/37* | 95.8% *23/24* | 10.8% *4/37* | 4.2% *1/24* | .85 |
| 10 | The CBI program helped me understand mathematics problem-solving better. | 87.9% *29/33* | 92.9% *26/28* | 12.1% *4/33* | 7.1% *2/28* | .42 |
| 11 | The CBI program made me like mathematics problem-solving better. | 77.1% *27/35* | 90.5% *19/21* | 22.9% *8/35* | 9.5% *2/21* | 1.59 |
| 12 | I came to like mathematics problem-solving after I tried the CBI program. | 69.7% *23/33* | 91.7% *22/24* | 30.3% *10/33* | 8.3% *2/24* | 4.04 |

Note. * Number of respondents/total sample size in each group.
Total sample size for each group varies because some of the students gave a "describes me somewhat" response so these data were not used in the Chi-square analyses.

The following results are based upon combining the two CBI groups (experimental and CBI control). It was found that students in both groups were interested in using the computer. On item #6, 77.2 percent of the students felt that the computer was their private tutor, and 90.5 percent responded that they were able to work at their own pace while taking the CBI program (item #7). With regard to the CBI program they received, 97 percent of the students rated it favorably.

97.0 percent of students in both groups felt that trying the CBI program was fun (item #5). On item #10, 90.2 percent of the students responded that the CBI program made them understand mathematics problem-solving better and 86.2 percent felt that the CBI program made them more interested in mathematics problem-solving (item #8). For item #11, 80.1 percent of the students perceived that the CBI program made them like problem-solving better. Moreover, 91.8 percent of the students liked the CBI program better than classroom instruction (item #9).

With respect to their liking of mathematics problem-solving, positive attitude changes, as indicated by responses on items #4 and 12, were observed. Students were asked whether they did not like mathematics problem-solving before they tried the CBI program. Interestingly, 68.5 percent of the students reported that they did not (item #4). Students showed a more positive attitude toward mathematics problem-solving after they had completed the CBI program. Noticeably, 78.9 percent of students reported that they came to like mathematics problem-solving better after completing the CBI program

(item #12). By developing a contingency table using items #4 and 12, it was found that 94.1 percent of the students who did not like mathematics problem-solving before they tried the CBI program came to like mathematics problem-solving after they had completed the program, $\chi^2$ (1, $\underline{N}$ = 47) = 17.39, $\underline{p} < .001$.

As mentioned earlier, participation in the CBI program, even though the experimental and CBI control groups did not receive the same type of CBI program, seems to have improved students' attitudes toward mathematics and mathematics problem-solving.

Due to the fact that the attitude measure did not include a comprehensive list of items, the results should be viewed with some caution. This refers specifically to research questions 7 and 8.

Comments on the CBI program

As part of the attitude survey, students in the experimental and CBI control groups were also asked for additional comments on the CBI program they received. Students in the experimental group received the CBI program with explicit strategies instruction, whereas those in the CBI control group received the CBI program without explicit strategies instruction. Of the 68 students (experiment = 39, CBI control = 29) who completed the survey, 62 students answered this additional question. In general, findings reported in this open-ended question paralleled findings reported for items #4 to 12.

for understanding mathematics problem-solving. They also stated that the CBI program allowed them to work at their own pace. Interestingly, some students in the experimental group described increased self-confidence in mathematics problem-solving, whereas comments from the CBI control group did not include statements about self-confidence. Generally, students described the CBI program as fun, self-paced, and helpful for understanding mathematics problem-solving.

The following comments illustrate their responses:

- Many students in the experimental and the CBI control groups described the CBI program as fun ($n = 24$):

- "I think it was fun because the computer helped me when I made mistakes, and keeps it organized (unlike I usually keep!). It gave you clear instructions and problems that were challenging." (experimental)

- "I liked Doehee's math lessons a lot. They were really fun and I learned a lot about math problems." (experimental)

- "I think that Doehee's math lesson was fun and it was more fun than normal math problems." (experimental)

- "Doehee's math lessons were fun and you could work on your own. She made problem-solving fun and easier." (CBI control)

- "I had lots of fun and I learned from my mistakes. It was really neat and interesting." (CBI control)

- Students in both groups felt that it was helpful for understanding mathematics problem-solving (n = 13):

- "Doehee's math lessons made me better at mathematics problem-solving. I enjoyed everything about it." (experimental)

- "It helps me understand math better. Thanks." (experimental)

- "It helped explain mathematics problem-solving better." (experimental)

- "I think Doehee's math lesson helped me and many others in this class understand problem-solving better." (CBI control)

- "I liked the math lessons very well. It helped me understand the problems better." (CBI control)

- They also felt that they were able to work at their own pace (n = 8):

- "Doehee's math lessons were great. I could work at my own speed and I understand it a lot." (experimental)

- "I think her lessons were fun and easy to understand. It was nice to be able to work at my own pace." (experimental)

- "Doehee's math lessons were great. I could work at my own speed and I understood it a lot. Keep up the good work." (experimental)

- "I think your program made me understand problem-solving and it allowed me to work at my own pace. It's not as confusing as normal classroom instructions." (CBI control)

- "I like it because I could work at my own pace and not HAVE to write down anything." (CBI control)

- Interestingly, some students in the experimental group expressed increased confidence in mathematics problem-solving (n = 2) and increased self-esteem

(n = 1), whereas students in the CBI control group did not mention self-confidence in their comments:

- "I think Doehee's math lessons helped me to understand problem-solving. If I can remember this information, I think I will do well in math. I feel more confident after using the math lesson." (experimental)

- "I liked Doehee's math program. I feel more confident in mathematics problem-solving now. It was fun working on the computer." (experimental)

- "..... before I thought I was really bad at math, now I like problem-solving more and I feel better about myself." (experimental)

- With regard to the speed of the CBI program, one student in the experimental group described the CBI program as slow, whereas one in the CBI control group described the CBI program as fast. Students' comments here reflect the speed of computers they used in the computer lab. Students in the experimental group used the Macintosh LC 575 whereas those in the CBI control group used the Macintosh 5400 Series. The following comments illustrate their responses:

- "I think it was very well organized, a little slow with something like that. Interesting, funny, and it had a good challenge and was very good." (experimental)

- "It was fun. It was easier because you didn't have to write so much. It was fast." (CBI control)

Summary

The findings of the attitude survey showed that a majority of students in the experimental and CBI control groups reported that they liked mathematics (74.2%-89.3%) and mathematics problem-solving (79.3%-80.0%), whereas only about 38% of those in the traditional control group did. With regard to liking of computer use, an overwhelming majority of students stated that they were interested in using the computer. With respect to students' attitudes toward the CBI program they received, there was no significant difference observed between the experimental and CBI control groups. The responses of those in the experimental and CBI control groups were overwhelmingly positive. They reported that the CBI program was fun (97.0%), self-paced (90.5%), and helpful for them to understand mathematics problem-solving better (90.2%). They also reported that they liked the CBI program better than classroom instruction (91.8%), and that the computer was like their private tutor (77.2%). Noticeably, 94.1 percent of the students who did not like mathematics problem-solving before they tried the CBI program

responded that they came to like mathematics problem-solving after they had completed the program, $\chi^2$ (1, $N$ = 47) = 17.39, $p$ < .001. Interestingly, some students in the experimental group reported increased self-confidence in mathematics problem-solving ($n$ = 2) and self-esteem ($n$ = 1), whereas such comments were not observed from students in the CBI control group. However, negative comments were made on the speed of the CBI program: one student in the experimental group described the CBI program as slow, whereas one in the CBI control group described the CBI program as fast.

# VI. Discussion

In this study, a component of instructional design was compared by administering two versions of a CBI program (i.e., Problem-Solving Strategies-CBI [PSS-CBI] and Control-CBI programs) to sixth-grade students at the beginning of the school year. Along with the experimental and CBI control groups, a traditional control group of sixth-grade students was also included in order to control for the effect of computer use. To examine the effects of explicit strategies instruction via computer, mathematics problem-solving performance (CTBS-MPS and MPST), self-prediction of performance (SPP), self-evaluation of performance (SEP), accuracy of self-prediction of performance (ASPP), accuracy of self-evaluation of performance (ASEP), perception of mathematics problem-solving processes (MPSPQ), and attitudes toward mathematics, mathematics problem-solving, and the CBI program they received were measured. This chapter discusses these research findings.

With respect to characteristics of high and low achieving students, there was evidence from pre- and post-test measures which leads to the conclusion that high achieving students showed higher

- mathematics problem-solving skills,
- self-prediction of performance (SPP),
- self-evaluation of performance (SEP), and
- perception of mathematics problem-solving processes

than low achieving students.

With respect to ASSP and ASEP, high achieving students did not surpass low achieving students in the pre-test measure; however, high achieving students showed more accurate estimates on their performance than low achieving students across the pre- and post-test periods. That is, with respect to change over time, high achieving students appeared to be more accurate in predicting and evaluating their performance than were low achieving students. Therefore, it is believed that high achieving students, in spite of the absence of feedback, became better able to monitor their own cognitive ability than did low achieving students.

These findings are consistent with previous research which has shown that high achieving students are more aware of their knowledge and problem-solving strategies and also were more successful in the task (Montague & Applegate, 1993; Montague, 1991; Montague, & Bos, 1990); high achieving students are more optimistic and accurate in their prediction and evaluation of performance (i.e., knowledge about cognition and regulation of cognition) (Slife et al., 1985).

With respect to the mathematics problem-solving measures, the experimental group achieved significantly higher mean gain scores on the CTBS-MPS measure than the CBI control and traditional control groups. However, a significant mean gain scores difference was not observed on the MPST, a researcher-designed measure. There are three plausible explanations for these results.

First, the CTBS-MPS, which was in multiple-choice form, covered single-step and multiple-step problems and focused simply on students' solutions, whereas the MPST, which was in open-ended form, covered exclusively process problems and focused more on students' procedural performance than on their solutions. Single-step and multiple-step problems can be solved by simply choosing the right operation or operations, whereas more complex, process problems can not be solved by simply adding, subtracting, multiplying, or dividing (Charles, Lester, & O'Daffer, 1987). The solutions of process problems involve more than computation. That is, process problems are solved using strategies such as Make a Table, Work Backwards, Draw a Chart or Picture, and Use Logical Reasoning. Therefore, it was felt that the MPST might have added a greater degree of difficulty than the CTBS-MPS due to the question complexity and the focus on the students' higher-level problem-solving processes, causing no significant effect of the explicit strategies instruction via computer on MPST scores. For this reason, it is possible that slight gain mean scores (pre-MPST to post-MPST) in the experimental group may have been significantly enhanced by a longer period of exposure to the explicit strategies instruction via computer (PSS-CBI program) because most of the contents of the CBI program required higher-level thinking skills.

Second, the PSS-CBI program for the experimental group took longer than the Control-CBI program due to the components of explicit strategies instruction embedded in the CBI program. It is possible, accordingly, that the

extended length of each lesson in the PSS-CBI program may have had an adverse effect on the outcome of the MPST measure. Perhaps, breaking each lesson into smaller sessions when working on each lesson in the PSS-CBI program might have improved students' achievement significantly.

Finally, due to conflicts in class scheduling, students in the experimental group were given the MPST after they had spent more than an hour taking the PSS-CBI program and the attitude survey, whereas those in the CBI control group were given the tests on separate days. Therefore, there could be a *fatigue* factor involved for some students in the experimental group. It is possible that some students in this group might have had a lack of engagement in the test-taking activity due to the elongated length of the PSS-CBI program. The extended testing length without a long enough break for the experimental group may have actually had a detrimental influence on the post-MPST measure.

With respect to SPP, SEP, ASPP, ASEP, and perception of mathematics problem-solving processes measures, it was expected that there would be significant mean score differences on these measures for achievement groups (high and low) among three groups (experimental, CBI control, and traditional control. Before trying to solve each problem on the MPST, students were asked to make predictions regarding their performance on the problem. The results showed that high achieving students were predominantly more optimistic about how well they would perform than low achieving students. With respect to SEP, high achieving students also

rated their performance as higher after solving each mathematics word problem on the MPST, indicating that they were generally more confident about their performance than low achieving students. With respect to ASPP and ASEP, high achieving students were more accurate in SPP and SEP than were low achieving students. Moreover, high achieving students were more aware of their mathematics problem-solving processes than low achieving students as shown on the results of the MPSPQ.

Since students' prediction and evaluation of performance "are probably due in part to task-related experience" (Pressley & Ghatala, 1989, p. 432), it was expected that these measures would be affected by their experience with explicit strategies instruction. However, students who were exposed to the explicit strategies instruction via computer (experimental group) failed to show significantly greater improvement on these measures than those who were exposed to the control-CBI program (CBI control group) or no computer environment (traditional control group). Perhaps, it takes a longer period of time than expected to enhance such metacognitive functions, as supported by Flavell, Miller, and Miller (1993). Since metacognitive skills are late in developing, trainable (Brown, 1980), and acquired gradually (Flavell et al., 1993), longer programs are more likely to be effective with young children than shorter ones (Simons, 1994). Therefore, it is understandable that the explicit strategies instruction via computer failed to show a significant improvement on these measures over a short period of time. It is suggested that explicit strategies instruction be used throughout the schooling years in

order to foster metacognitive functions.

With respect to attitudes toward computer use, it was observed that there was no significant difference among the three groups (experimental, CBI control, and traditional control). That is, an overwhelming majority of students across the groups expressed that they were interested in using the computer. This finding was consistent with Rowe's (1993) study which found that nearly all students were enthusiastic about using computers.

With respect to attitudes toward mathematics and mathematics problem-solving, it was observed that there were significant differences on attitudes toward mathematics and mathematics problem-solving among the three groups (experimental, CBI control, and traditional control). The results showed that a large number of the students who received the CBI program, with or without strategies instruction, enjoyed mathematics and mathematics problem-solving, whereas only a relatively small number of students who did not receive any type of CBI program did. Findings further suggest that students in the experimental and CBI control groups altered their views of mathematics and mathematics problem-solving: Regardless of whether they received explicit strategies instruction, students who received the CBI program (PSS-CBI or Control-CBI) came to like mathematics and mathematics problem-solving better than students who did not receive any type of CBI program. Participating in the CBI program appears to have improved students' attitudes toward mathematics and mathematics problem-solving. These findings are supported by Funkhouser's (1993) and Okolo's

(1992) studies which also found that attitudes were improved through the use of computers. Funkhouser's (1993) study demonstrated that students had increasingly positive attitudes and improved performance in mathematics; Okolo's (1992) study indicated that computers improved achievement motivation as well as academic skills.

With respect to students' attitudes toward the CBI program they received, there were no significant differences observed between the experimental and CBI control groups. Virtually all students in these groups liked the CBI program better than classroom instruction. Although the responses did not confirm that the CBI program is significantly better than classroom instruction, it does give a good indication of how a significant number of students feel about the CBI program. A majority of students responded that they felt that the computer was their tutor, and that the CBI program was fun, self-paced, and helpful for them to understand mathematics problem-solving better. More interestingly, a large number of students showed a more positive attitude change in their liking of mathematics problem-solving after they had completed the CBI program as found from the Likert-scale as well as in the open-ended comments section of the attitude survey. These findings suggest that a CBI program can play a valuable role in improving students' attitudes toward mathematics problem-solving.

The only negative comments regarding the CBI program were concerned with the speed of the program based on the type of computer they

used: that is, the slowness for the experimental group and the speed for the CBI control group. Specifically, students in the experimental group used the Macintosh LC 575, whereas those in the CBI control group used the Macintosh 5400 Series. The difference in the quality of the computer laboratory facilities between the two CBI groups (experimental and CBI control) might have had a detrimental effect on the performance of the students in the experimental group. Students in this group sometimes showed frustration due to the system errors and slowness of their computers. Despite the fact that they had several obstacles to overcome, the experimental group still had, as reported earlier, a positive attitude toward computer use in general, as well as positive comments about the CBI program they received.

Interestingly, a few students in the experimental group described increased self-confidence in mathematics problem-solving and increased self-esteem, whereas comments from the CBI control group did not include such statements. It appears that introducing explicit strategies instruction via computer (PSS-CBI program) may have a positive impact on some students' self-esteem and self-confidence, even though there was no significant difference observed on the ratings on the CBI programs between the experimental (PSS-CBI) and CBI control (Control-CBI) groups. It is possible, however, that were self-esteem and self-confidence ratings included on the Likert-scale, a significant difference between the two groups on these affective attributes may have been found.

Therefore, it can be concluded that having mathematics problem-

solving experience as well as experience with explicit strategies instruction via computer can enhance one's positive attitudes toward mathematics and/or mathematics problem-solving, even improving one's self-esteem and self-confidence. Furthermore, problem-solving experience via computer is apt to build a good base of transfer to promote learning of general problem-solving skills, since affective attributes (e.g., attitude toward mathematics) appear to have a powerful influence on one's engagement in related learning activities and consequently in one's achievement on those activities (Carr & Jessup, 1995; Lester, 1990; McLeod, 1985; Silver, 1985; Tyler & Vasu, 1995).

## Observations To Be Considered

Two additional observations were made regarding school climate and previous experience in problem-solving strategies instruction. First, with respect to the school climate factor involved in this study, the school environment for the experimental group was found to be adequate. However, the CBI control group school was observed to be exceptional. That is, students in the CBI control group were more well-behaved, listened more attentively, and were more focused on the task at hand. As well, school climate in the CBI control group was remarkably warm and supportive. This type of school environment could "encourage students to participate and become academically involved; this increased academic involvement would in turn be accompanied by higher achievement" (Voelkl, 1995). Therefore, it is assumed that this is the reason students in the CBI control group were

more focused during the study, whereas those in the experimental group were somewhat distracted. Had the experimental group been exposed to a similar environment as the CBI control group, experimental effects may have emerged.

No quantitative data are available to corroborate this claim since school climate was not systematically measured; however, through observations made during the study, it was felt that school climate may have affected students' learning behaviors. Some students in the experimental group appeared to have a lack of engagement in the learning activity due to their less cohesive classroom environment and consequently their attitudes toward learning and their behavior. Even though classroom environment does not affect students' achievement directly, it may affect students' learning attitude and behavior before it affects their academic achievement (Cheng, 1994).

Second, in order to minimize participating students' knowledge gap within the subject area, this study was conducted at the beginning of the school year. However, it was found, after talking to one of the teachers in the CBI control group, that 50% of the students in this group had intensive instruction in mathematics problem-solving strategies in the previous school year. In contrast, students in the experimental group were not very familiar with mathematics problem-solving strategies instruction. Therefore, it is possible that students' past experience in learning mathematics problem-solving strategies might have affected the results.

# VII. Conclusion

This study explored the potential of explicit strategies instruction via computer on sixth-grade students' mathematics problem-solving skills. Specifically, the study examined whether explicit strategies instruction via computer affects students on the following variables: mathematics problem-solving achievement (CTBS-MPS and MPST); attitudes toward mathematics, mathematics problem-solving, and computer use; attitudes toward the CBI program which they received; perception of mathematics problem-solving processes; self-prediction of performance (SPP); self-evaluation of performance (SEP); accuracy of self-prediction of performance (ASPP); and accuracy of self-evaluation of performance (ASEP). This final chapter is divided into three subsections: summary of findings, recommendations for future research, and limitations and delimitations.

## Summary of Findings

This study found that explicit strategies instruction via computer has a beneficial effect on the CTBS-MPS scores. Students who received the PSS-CBI program (experimental group) significantly outperformed those who received the Control-CBI program (CBI control group) and those who did not receive any type of CBI program (traditional control group). However, with regard to a researcher-designed measure, the MPST, there were no significant differences observed among the three groups (experimental, CBI control, and

traditional control). Process problems in the MPST were in open-ended form requiring the use of strategies such as Make a Table, Work Backwards, or Use Logical Reasoning, as well as computation skills to reach the solutions. In contrast, single-step and multiple-step problems in the CTBS-MPS were in multiple-choice form and required merely selecting the operation or operations necessary to choose the right answers. A possible explanation for this disparity in the results of the CTBS-MPS and MPST measures is that it may take a longer period of exposure to explicit strategies instruction via computer (PSS-CBI program) to obtain a significant improvement in students' performance on process problems. Therefore, future research should be conducted over a longer period of time in order to verify the effectiveness of explicit strategies instruction via computer.

This study also found that students in either experimental (PSS-CBI program) or CBI control (Control-CBI program) groups showed more positive attitudes toward mathematics and mathematics problem-solving than those who did not receive any type of CBI program. However, there were no significant differences on attitudes toward the mathematics and mathematics problem-solving between the experimental and CBI control groups. It was observed, from the attitude questionnaire on the CBI program, that a large number of students who were exposed to the CBI program (PSS-CBI or Control-CBI) showed positive attitude changes in mathematics and mathematics problem-solving: They came to like mathematics and mathematics problem-solving better after they had completed the CBI

program. A majority of students in the experimental and CBI control groups also stated that the CBI program which they received was fun, self-paced, and helpful for them to understand mathematics problem-solving better. Moreover, they stated that they liked the CBI program better than classroom instruction. Interestingly, some students who were exposed to explicit strategies instruction via computer reported increased self-confidence and self-esteem, whereas such statements were not reported from students in the CBI control group. These positive findings from the two CBI groups appear to demonstrate that the CBI program has a promising future for improving students' attitudes toward mathematics and mathematics problem-solving.

In addition to the findings mentioned above, this study also found that there were no significant positive changes across the three groups in perceptions of mathematics problem-solving processes, SPP, SEP, ASPP, and ASEP. Perhaps, it takes a much longer period of exposure to explicit strategies instruction to obtain positive changes in students' metacognitive functions. Therefore, it calls for future research to ascertain whether there will be a beneficial effect of the PSS-CBI program on these metacognitive functions when this study is replicated using a longer period of time. Further recommendations for future research on explicit strategies instruction via computer are given below.

## Recommendations for Future Research

- The findings from this study suggest that explicit strategies instruction via computer (PSS-CBI program) was successful in improving sixth-grade students' performance on single-step and multiple-step problems. Students who were exposed to the PSS-CBI program significantly improved their scores on the CTBS-MPS. However, the PSS-CBI program failed to show a significant effect on students' performance on process problems in the MPST, which seemed to be more complex than single-step or multiple-step problems. One of the drawbacks of this study was the short period of exposure to the explicit strategies instruction via computer which corresponds to the first phase of the SPELT approach: Direct Teaching of Strategies. Since metacognitive skills develop gradually, a longitudinal study should be conducted through the systematic transfer and generalization of strategies instruction which ultimately leads to self-generated learning and thinking which corresponds to the second and third phases of SPELT: Maintenance and Generalization of Strategies and Strategy Generation by Students (Marfo, Mulcahy, Peat, Andrews, & Cho, 1991; Mulcahy, 1991; Mulcahy, Marfo, Peat, & Andrews, 1987).

- This could be done, for example, using a larger sample of students to ascertain whether their learning experience in explicit strategies instruction via computer has a beneficial impact on their performance in more complex mathematics word problems as well as metacognitive skills

(e.g., self-prediction of performance, self-evaluation of performance, and perceptions of mathematics problem-solving processes).

- This study did not include a maintenance phase to examine if a long-term effect of explicit strategies instruction via computer would occur. Therefore, it is uncertain whether a positive effect of explicit strategies instruction via computer would be sustained over a longer period of time. Further study needs to include a follow-up session.
- The purpose of this study was to explore the potential of explicit strategies instruction via computer. However, in order to examine whether the combination of the PSS-CBI program and classroom instruction is even more beneficial for enhancing students' mathematics problem-solving skills than the PSS-CBI program alone, implementing a combination of the PSS-CBI program and classroom instruction is recommended for future research.
- The results of the CTBS-MPS measure indicate that the SPIDER approach can play an important role in teaching mathematics problem-solving skills by involving students in problem-solving processes using a step-by-step method. Therefore, it would be interesting to see whether this process-oriented approach, SPIDER, could be transferable to other areas of the curriculum.
- Some students who were exposed to explicit strategies instruction via computer (PSS-CBI program) reported increased self-confidence in mathematics problem-solving and self-esteem; however, such comments

were not observed from those who received the Control-CBI program. In order to explore the effect of explicit strategies instruction via computer on students' self-confidence and self-esteem, future research should include a systematic measure of these affective attributes.

- School climate may not directly affect the learning outcome; however, it affects students' attitudes and behavior, which have an enormous impact on maintaining motivation for learning, and subsequently affects their learning outcome. Therefore, a systematic measure of school climate should be included for future study in order to clarify any anomalous results.

## Limitations and Delimitations

One of the pitfalls of implementing new instruction (i.e., problem-solving strategies instruction via computer) into classroom settings is that there is the possibility of the results being contaminated by what is known as the novelty effect. In spite of this, it was believed that the length of the investigation and the fact that it was structured into the regular classroom should have minimized the novelty effect. For this reason, the CBI control group was also exposed to a computer-based instruction (CBI) setting.

It appeared that a majority of the participating students and teachers in the experimental and CBI control groups were enthusiastic about using the CBI program. Therefore, it is possible that the Hawthorne effect may have influenced the results of this study. In addition, the researcher was not able to

be present during their regular classroom instruction. Thus, it was not certain whether the students might have had mathematics problem-solving strategies instruction during that time.

With respect to the attitude measure, given that it did not include a comprehensive set of items, the results should be viewed with some caution. This refers to the attitude measure of mathematics, mathematics problem-solving, computer use, and the CBI program.

The test items were limited to whole numbers, in order to be consistent with the sixth-grade mathematics curriculum. The content validity of the items used in the CBI lessons and pre- and post-tests were determined by content experts such as mathematics teachers in schools.

Furthermore, since the experiment was limited to sixth-grade students following a general mathematics curriculum, the results can be generalized only to sixth-grade students.

## References

Alley, G., & Deshler, D. (1979). Learning strategies: An approach for the learning disabled adolescent, Teaching the learning disabled adolescent: Strategies and methods (pp. 11-56). Denver, CO: Love.

Artzt, A. F., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem-solving in small groups. Cognition and Instruction, 9(2), 137-175.

Baldrey, S. (1991). Microcomputer applications for people with learning difficulties. In A. Ager (Ed.), Microcomputers and clinical psychology: Issues, applications and future developments (pp. 139-154). New York, NY: John Wiley & Sons.

Bennett, J. P. (1992). Computers in mathematics education: A "not really for the researcher" review of recent unique findings. School Science & Mathematics, 92(1), 38-39.

Billingsley, B. S., & Wildman, T. M. (1990). Facilitating reading comprehension in learning disabled students: Metacognitive goals and instructional strategies. Remedial and Special Education (RASE), 11(2), 18-31.

Bloom, B. S., & Broder, L. J. (1950). Problem-solving processes of college students. Chicago, IL: University of Chicago.

Borkowski, J. G. (1992). Metacognitive theory: A framework for teaching literacy, writing, and mathematics skills. Journal of Learning Disabilities, 25(4), 253-257.

Borkowski, J. G., Carr, M., Rellinger, E., & Pressley, M. (1990). Self-regulated cognition: Interdependence of metacognition, attributions, and self-esteem. In B. F. Jones & L. Idol (Eds.), Dimensions of thinking and cognitive instruction (pp. 53-92). Hillsdale, NJ: Lawrence Erlbaum.

Borkowski, J. G., Estrada, M. T., Milstead, M., & Hale, C. A. (1989). General problem-solving skills: Relations between metacognition and strategic processing. Learning Disability Quarterly, 12, 57-70.

Bransford, J. D., & Stein, B. S. (1984). The IDEAL problem solver: A guide for improving thinking, learning, and creativity. New York, NY: W. H. Freeman.

Brown, A. (1987). Metacognition, executive control, self-regulation, and other more mysterious mechanisms. In F. E. Weinert & R. H. Kluwe (Eds.), Metacognition, Motivation, and Understanding (pp. 65-116). Hillsdale, NJ: Lawrence Erlbaum.

Brown, A. L. (1980). Metacognitive development and reading. In J. J. Spiro, B. C. Bruce, & W. F. Brewer (Eds.), Theoretical issues in reading comprehension: Perspective from cognitive psychology, linguistics, artificial intelligence, and education (pp. 453-481). Hillsdale, NJ: Lawrence Erlbaum.

Butterfield, E. C. (1994). Metacognition. In R. J. Sternberg (Ed.), Encyclopedia of human intelligence (pp. 725-732). NY, NY: MacMillan.

Callahan, L. G., & Garofalo, J. (1987). Metacognition and school mathematics. Arithmetic Teacher, 34(5), 22-23.

Cardelle-Elawar, M. (1992). Effects of teaching metacognitive skills to students with low mathematics ability. Teaching and Teacher Education, 8(2), 109-121.

Cardelle-Elawar, M. (1995). Effects of metacognitive instruction on low achievers in mathematics problems. Teaching and Teacher Education, 11(1), 81-95.

Cardelle-Elawar, M., & Wetzel, K. (1995). Students and computers as partners in developing students' problem-solving skills. Journal of Research on Computing in Education, 27(4), 387-401.

Carr, M., & Jessup, D. L. (1995). Cognitive and metacognitive predictors of mathematics strategy use. Learning and Individual Differences, 7(3), 235-247.

Case, L. P., Harris, K. R., & Graham, S. (1992). Improving the mathematical problem-solving skills of students with learning disabilities: Self-regulated strategy development. The Journal of Special Education, 26(1), 1-19.

Cawley, J. F., & Miller, J. H. (1986). Selected views on metacognition, arithmetic problem solving, and learning disabilities. Learning Disabilities Focus, 2(1), 36-48.

Charles, R., & Lester, F. (1982). Teaching problem solving: What, why, and how. Palo Alto, CA: Dale Seymour.

Charles, R., Lester, F., & O'Daffer, P. (1987). How to evaluate progress in problem solving. Reston, VA: National Council of Teachers of Mathematics.

Charles, R. I., & Lester, F. K. (1984). An evaluation of a process-oriented instructional program in mathematical problem solving in grades 5 and 7. Journal for Research in Mathematics Education, 15(1), 15-34.

Cheng, Y. C. (1994). Classroom environment and student affective performance: An effective profile. Journal of Experimental Education, 62(3), 221-239.

Coltharp, H. I. K. (1990). The effect of metacognitive strategies upon mathematical problem-solving ability. Unpublished doctoral dissertation, University of Missouri, Kansas.

Day, V. P., & Elksnin, L. K. (1994). Promoting strategic learning. Intervention in School and Clinic, 29(5), 262-270.

Derry, S. J. (1990). Remediating academic difficulties through strategy training: Acquisition of useful knowledge. Remedial and Special Education, 11(8), 19-31.

Derry, S. J., & Kellis, A. (1986). A prescriptive analysis of low-ability problem-solving behavior. Instructional Science, 15, 49-65.

Dominowski, R. L. (1992). Problem solving and metacognition. In K. J. Gilhooly, M. T. G. Keane, R. H. Logie, & G. Erdos (Eds.), Lines of thinking: Reflections on the psychology of thought (Vol. 2, pp. 313-328). New York, NY: John Wiley & Sons.

Dudley-Marling, C., & Owston, R. D. (1988). Using microcomputers to teach problem solving: A critical review. Educational Technology, 28(7), 27-33.

English, L. D., & Halford, G. S. (1995). Mathematics education: Models and processes. Hillsdale, NJ: Lawrence Erlbaum.

Flavell, J. (1976). Metacognitive aspects of problem solving. In L. B. Resnick (Ed.), The nature of intelligence (pp. 231-236). Hillsdale, NJ: Lawrence Erlbaum.

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-development inquiry. American Psychologist, 34(10), 906-911.

Flavell, J. H. (1985). Middle childhood and adolescence, Cognitive Development (2nd Edition ed., pp. 74-113). Englewood Cliffs, NJ: Prentice-Hall.

Flavell, J. H. (1987). Speculations about the nature and development of metacognition. In F. E. Weinert & R. H. Kluwe (Eds.), Metacognition, motivation, and understanding (pp. 21-30). Hillsdale, NJ: Lawrence Erlbaum.

Flavell, J. H., Miller, P. H., & Miller, S. A. (1993). Cognitive development. (3rd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Funkhouser, C. (1993). The influence of problem-solving software on student attitudes about mathematics. Journal of Research on Computing in Education, 25(3), 339-346.

Garofalo, J. (1985). Mathematical performance: What's in it for developmental educators? Research and Teaching in Developmental Education (RTDE), 2(1), 24-27, 30-33.

Garofalo, J. (1986). Metacognitive knowledge and metacognitive process: Important influences on mathematical performance. Research and Teaching in Developmental Education (RTDE), 2(2), 34-39.

Garofalo, J. (1993). Mathematical problem preferences of meaning-oriented and number-oriented problem solvers. Journal for the Education of the Gifted, 17(4), 26-40.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. Journal for Research in Mathematics Education, 16(3), 163-176.

Geisert, P. G., & Futrell, M. K. (1995). Teachers, computers, and curriculum: Microcomputers in the classroom. (2nd ed.). Needham Heights, MA: Allyn and Bacon.

Goldman, S. R. (1989). Strategy instruction in mathematics. Learning Disabilities Quarterly, 12(Winter), 43-55.

Hart, L. C. (1993). Some factors that impede or enhance performance in mathematical problem solving. Journal for Research in Mathematics Education, 24, 167-171.

Hembree, R. (1992). Experiments and relational studies in problem solving: A meta-analysis. Journal for Research in Mathematics Education, 23(3), 242-273.

Hofmeister, A. M. (1990). Intelligent computer-aided instruction of study and metacognitive strategies. Final report. (ED 341 206). Logan, UT: Utah State University.

Howson, A. G., & Wilson, B. (1986). Classrooms and teachers in the 1990s. In A. G. Howson & B. J. Wilson (Eds.), School Mathematics in the 1990s (Vol. 2, pp. 75-82). Cambridge, MA: Cambridge University.

Jones, B. F., Palincsar, A. S., Ogle, D. S., & Carr, E. G. (1987). Strategic teaching and learning: Cognitive instruction in the content areas. Alexandria, VA: Association for Supervision and Curriculum Development in cooperation with the North Central Regional Educational Laboratory.

Jussel, M. R. (1993). Teaching problem solving strategies with and without the computer. In J. H. Wiebe (Ed.), Computer tools and problem solving in mathematics (pp. 31-34). Wilsonville, OR: Franklin, Beedle, & Associates.

Kaput, J. J., & Thompson, P. W. (1994). Technology in mathematics education research: The first 25 years in the JRME. Journal of Research in Mathematics Education, 25(6), 676-684.

Keller, C. E., & Lloyd, J. W. (1989). Cognitive training: Implications for arithmetic instruction. In J. N. Hughes & R. J. Halls (Eds.), Cognitive behavioral psychology in the schools: A comprehensive handbook (pp. 280-304). N.Y.: Guildford.

Kersh, M. E., & McDonald, J. (1991). How do I solve thee? Let me count the ways. Arithmetic Teacher, 39(2), 38-41.

Kilpatrick, J. (1985). Reflection and recursion. Educational Studies in Mathematics, 16, 1-26.

Kluwe, R. H. (1987). Executive decisions and regulation of problem solving behavior. In F. E. Weinert & R. H. Kluwe (Eds.), Metacognition, motivation, and understanding (pp. 31-64). Hillsdale, NJ: Lawrence Erlbaum.

Kulik, J. A. (1994). Meta-analytic studies of findings on computer-based instruction. In E. L. Baker & H. F. O'Neil (Eds.), Technology assessment in education and training (pp. 9-33). Hillsdale, NJ: Lawrence Erlbaum.

Lester, F. K. (1985). Methodological considerations. In E. A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives (pp. 41-70). Hillsdale, NJ: Lawrence Erlbaum.

Lester, F. K. (1994). Musings about mathematical problem-solving research: 1970-1994. Journal for Research in Mathematics Education, 25(6), 660-675.

Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). Self-confidence, interest, beliefs, and metacognition: Key influences on problem-solving behavior. In D. B. McLeod & V. M. Adams (Eds.), Affect and mathematical problem solving: A new perspective (pp. 75-88). New York, NY: Spring-Verlag.

Lin, A., Podell, D. M., & Tournaki-Rein, N. (1994). CAI and the development of automaticity in mathematics skills in students with and without mild mental handicaps. Computers in the Schools, 11(1), 43-58.

Locke, E. T., & Abbey, D. E. (1989). A unique equation: Learning strategies + generalization = success. Academic Therapy, 24(5), 569-575.

Manning, B. H. (1991). Cognitive self-instruction for classroom processes. Albany, NY: State University of New York.

Marfo, K., Mulcahy, R. F., Peat, D., Andrews, J., & Cho, S. (1991). Teaching cognitive strategies in the classroom: A content-based instructional model. In R. F. Mulcahy, R. H. Short, & J. Andrews (Eds.), Enhancing learning and thinking (pp. 67-95). New York, NY: Praeger.

Masingila, J. O., & Moellwald, F. E. (1993). Using Polya to foster a classroom environment for real-world problem solving. School Science and Mathematics, 93(5), 245-249.

McCoy, L. P. (1994). Mathematical problem-solving processes of elementary male and female students. School Science and Mathematics, 94(5), 266-270.

McLeod, D. B. (1988). Affective issues in mathematical problem solving: Some theoretical considerations. Journal for Research in Mathematics Education, 19(2), 134-141.

McLeod, D. B. (1989). Affect and mathematical problem solving. In D. B. McLeod & V. M. Adams (Eds.), Affect and mathematical problem solving: A new perspective (pp. 20-36). New York, NY: Springer-Verlag.

McLeod, D. B. (1994). Research on affect and mathematics learning in the JRME: 1970 to present. Journal for Research in Mathematics Education, 25(6), 637-647.

Meltzer, L. J. (1984). Cognitive assessment and the diagnosis of learning problems. In M. D. Levine & P. Satz (Eds.), Middle childhood: Development and dysfunction (pp. 131-152). Baltimore, MD: University Park.

Meltzer, L. J., Solomon, B., Fenton, R., & Levine, M. D. (1989). A developmental study of problem-solving strategies in children with and without learning difficulties. Journal of Applied Developmental Psychology, 10(2), 171-193.

Miller, S. P., & Mercer, C. D. (1993). Mnemonics: Enhancing the math performance of students with learning difficulties. Intervention in School and Clinic, 29(2), 78-82.

Montague, M. (1988). Strategy instruction and mathematical problem solving. Reading, Writing, and Learning Disabilities, 4, 275-290.

Montague, M. (1991). Gifted and learning-disabled gifted students' knowledge and use of mathematical problem-solving strategies. Journal for the Education of the Gifted, 14(4), 393-411.

Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. Journal of Learning Disabilities, 25(4), 230-248.

Montague, M., & Applegate, B. (1993). Mathematical problem-solving characteristics of middle school students with learning disabilities. The Journal of Special Education, 27(2), 175-201.

Montague, M., & Bos, C. (1986). The effect of cognitive strategy training on verbal math problem solving performance of learning disabled adolescents. Journal of Learning Disabilities, 19, 26-33.

Montague, M., & Bos, C. (1990). Cognitive and metacognitive characteristics of eighth grade students' mathematical problem solving. Learning and Individual, 2(3), 371-388.

Mulcahy, R., Marfo, K., Peat, D., & Andrews, J. (1987). SPELT: A strategies program for effective learning/thinking - Inservice edition. Edmonton, AB: Department of Educational Psychology, University of Alberta.

Mulcahy, R. F. (1987). Your thoughts about problem solving. Edmonton, AB: Department of Educational Psychology, University of Alberta.

Mulcahy, R. F. (1991). Developing autonomous learners. The Alberta Journal of Educational Research, 37(4), 385-397.

Nandi, S. L. (1993). Metacognition: A developmental study of intellectually gifted and average children. Unpublished doctoral dissertation, Carleton University, Ottawa, Ontario, Canada.

National Council of Teachers of Mathematics (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

National Council of Teachers of Mathematics (1996). A perspective on reform in mathematics and science education. Columbus, OH: Clearinghouse.

Okamoto, M., & Kitao, N. (1992). The role of metacognitive knowledge and aptitude in arithmetic problem solving. Psychologia, 35, 164-172.

Okolo, C. M. (1992). The effects of computer-based attribution retraining on the attributions, persistence, and mathematics computation of students with learning disabilities. Journal of Learning Disabilities, 25(5), 327-334.

Paris, S. G., & Oka, E. R. (1986). Self-regulated learning among exceptional children. Exceptional Children, 53(2), 103-108.

Paris, S. G., & Winograd, P. (1990). Promoting metacognition and motivation of exceptional children. Remedial and Special Education (RASE), 11(6), 7-15.

Polya, G. (1957). How to solve it. Princeton, NJ: Princeton University.

Pressley, M., & Ghatala, E. (1989). Metacognitive benefits of taking a test for children and young adolescents. Journal of Experimental Child Psychology, 47, 430-450.

Pressley, M., Levin, J. R., Ghatala, E. S., & Ahmad, M. (1987). Test monitoring in young grade school children. Journal of Experimental Child Psychology, 43(1), 96-111.

Ralston, A. (1993). Understanding mathematics: The impact of technology. In R. B. Davis & C. A. Maker (Eds.), Schools, mathematics, and the world of reality (pp. 79-92). Boston, MA: Allyn and Bacon.

Romberg, T. A., & Carpenter, T. P. (1986). Research on teaching and learning mathematics: Two disciplines of scientific inquiry. In M. C. Wittrock (Ed.), Handbook of research on teaching (3rd ed., pp. 850-873). New York, NY: MacMillan.

Rowe, H. A. H. (1993). Learning with personal computers. Victoria, Australia: Australian Council for Education Research.

Salomon, G., Globerson, T., & Guterman, E. (1989). The computer as a zone of proximal development: Internalizing reading-related metacognitions from a reading partner. Journal of Educational Psychology, 89(4), 620-627.

Schoenfeld, A. H. (1985). Explicit heuristic training in problem solving, Mathematical Problem Solving (pp. 189-215). Orlando, FL: Academic.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), Cognitive Science and Mathematics Education (pp. 189-216). Hillsdale, NJ: Lawrence Erlbaum.

Siegler, R. S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), The Origins of Cognitive Skills: The Eighteenth Annual Carnegie Symposium on Cognition (pp. 229-294). Hillsdale, NJ: Lawrence Erlbaum.

Silver, E. A. (1982). Knowledge organization and mathematical problem solving. In F. K. Lester & J. Garofalo (Eds.), Mathematical Problem Solving: Issues in Research (pp. 15-25). Philadelphia, PA: Franklin.

Silver, E. A. (1985). Research on teaching mathematical problem solving: Some underrepresented themes and needed directions. In E. A. Silver (Ed.), Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives (pp. 247-266). Hillsdale, NJ: Lawrence Erlbaum.

Silver, E. A., & Marshall, S. P. (1990). Mathematical and scientific problem solving: Findings, issues, and instructional implications. In B. F. Jones & L. Idol (Eds.), Dimensions of Thinking and Cognitive Instruction (pp. 265-290). Hillsdale, NJ: Lawrence Erlbaum.

Simons, P. R.-J. (1994). Metacognitive strategies, teaching, and testing. In T. Husen & T. N. Postlethwaite (Eds.), The international encyclopedia of education (2nd ed., Vol. 7, pp. 3788-3792). Great Britain: Pergamon.

Slife, B. D., Weiss, J., & Bell, T. (1985). Separability of metacognition and cognition: Problem solving in learning disabled and regular students. Journal of Educational Psychology, 77(4), 437-445.

Sternberg, R. J. (1982). Reasoning, problem solving, and intelligence. In R. J. Sternberg (Ed.), Handbook of Human Intelligence (pp. 227-307). New York, NY: Cambridge University.

Suydam, M. (1978). Problem solving packet (ED 164 328).

Taback, S. F. (1992). Enhancing the teaching of mathematical problem solving. School Science and Mathematics, 92(5), 253-256.

Tringa, P. K., & Lipitakis, E. A. (1995). A study of teaching mathematical concepts with computers. International Journal of Mathematics Education in Science and Technology, 26(4), 473-488.

Tyler, D. K., & Vasu, E. S. (1995). Locus of control, self-esteem, achievement motivation, and problem-solving ability: LogoWriter and simulations in the fifth-grade classroom. Journal of Research on Computing in Education, 28(1), 98-120.

Van Reusen, A. K., & Head, D. N. (1994). Cognitive and metacognitive interventions: Important trends for teachers of students who are visually impaired. Re:View, 25(4), 153-162.

Voelkl, K. E. (1995). School warmth, student participation, and achievement. Journal of Experimental Education, 63(2), 127-138.

Woodward, J., Carnine, D., & Gersten, R. (1988). Teaching problem solving through computer simulations. American Educational Research Journal, 25(1), 72-86.

Zbiek, R. M. (1989). Metacognitive skills, problem solving and the mathematics classroom (ERIC 309 989).

# Appendix A

Tables A1 to A7

Table A1

Analysis of Variance for the CTBS-MPS with Repeated Measures on Time Factor

******Analysis of Variance--design 1******

Tests of Between-Subjects Effects.

Tests of Significance for T1 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 1979.86 | 99 | 20.00 | | |
| Group | 10.36 | 2 | 5.18 | .26 | .772 |
| Ability | 5460.78 | 1 | 5460.78 | 273.06 | .000 |
| Group BY Ability | 102.57 | 2 | 51.28 | 2.56 | .082 |

******Analysis of Variance--design 1******

Tests involving 'TIME' Within-Subject Effect.

Tests of Significance for T2 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 598.10 | 99 | 6.04 | | |
| TIME | 67.25 | 1 | 67.25 | 11.13 | .001 |
| Group BY TIME | 43.63 | 2 | 21.82 | 3.61 | .031 |
| Ability BY TIME | 39.36 | 1 | 39.36 | 6.52 | .012 |
| Group BY Ability BY TIME | 20.67 | 2 | 10.34 | 1.71 | .186 |

Table A2

Analysis of Variance for the MPST with Repeated Measures on Time Factor

******Analysis of Variance -- design 1******

Tests of Between-Subjects Effects.

Tests of Significance for T1 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 139.96 | 99 | 1.41 | | |
| Group | 2.50 | 2 | 1.25 | .88 | .416 |
| Ability | 64.32 | 1 | 64.32 | 45.49 | .000 |
| Group BY Ability | .55 | 2 | .28 | .19 | .823 |

******Analysis of Variance -- design 1******

Tests involving 'TIME' Within-Subject Effect.

Tests of Significance for T2 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 91.00 | 99 | .92 | | |
| TIME | 79.98 | 1 | 79.98 | 87.02 | .000 |
| Group BY TIME | 1.70 | 2 | .85 | .92 | .401 |
| Ability BY TIME | .01 | 1 | .01 | .01 | .937 |
| Group BY Ability BY TIME | 2.57 | 2 | 1.29 | 1.40 | .251 |

Table A3

Analysis of Variance for Self-Prediction of Performance (SPP) with Repeated Measures on Time Factor

******Analysis of Variance--design 1******

Tests of Between-Subjects Effects.

Tests of Significance for T1 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 32.61 | 73 | .45 | | |
| Group | 1.13 | 2 | .57 | 1.27 | .288 |
| Ability | 5.95 | 1 | 5.95 | 13.32 | .000 |
| Group BY Ability | .18 | 2 | .09 | .20 | .819 |

******Analysis of Variance--design 1******

Tests involving 'TIME' Within-Subject Effect.

Tests of Significance for T2 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 15.84 | 73 | .22 | | |
| TIME | 7.14 | 1 | 7.14 | 32.91 | .000 |
| Group BY TIME | .01 | 2 | .01 | .03 | .973 |
| Ability BY TIME | .24 | 1 | .24 | 1.10 | .297 |
| Group BY Ability BY TIME | .97 | 2 | .49 | 2.24 | .114 |

Table A4

Analysis of Variance for Self-Evaluation of Performance (SEP) with Repeated Measures on Time Factor

******Analysis of Variance -- design 1******

Tests of Between-Subjects Effects.

Tests of Significance for T1 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 37.20 | 83 | .45 | | |
| Group | 1.34 | 2 | .67 | 1.50 | .230 |
| Ability | 5.17 | 1 | 5.17 | 11.54 | .001 |
| Group BY Ability | .18 | 2 | .09 | .20 | .817 |

******Analysis of Variance -- design 1******

Tests involving 'TIME' Within-Subject Effect.

Tests of Significance for T2 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 14.33 | 83 | .17 | | |
| TIME | 2.25 | 1 | 2.25 | 13.05 | .001 |
| Group BY TIME | .06 | 2 | .03 | .17 | .845 |
| Ability BY TIME | .26 | 1 | .26 | 1.52 | .222 |
| Group BY Ability BY TIME | .16 | 2 | .08 | .45 | .639 |

Table A5

Analysis of Variance for Accuracy of Self-Prediction of Performance (ASPP) with Repeated Measures on Time Factor

******Analysis of Variance--design 1******

Tests of Between-Subjects Effects.

Tests of Significance for T1 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 1.41 | 68 | .02 | | |
| Group | .03 | 2 | .01 | .72 | .491 |
| Ability | .15 | 1 | .15 | 7.27 | .009 |
| Group BY Ability | .01 | 2 | .01 | .27 | .764 |

******Analysis of Variance--design 1******

Tests involving 'TIME' Within-Subject Effect.

Tests of Significance for T2 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 1.09 | 68 | .02 | | |
| TIME | .04 | 1 | .04 | 2.42 | .125 |
| Group BY TIME | .00 | 2 | .00 | .04 | .965 |
| Ability BY TIME | .05 | 1 | .05 | 2.88 | .095 |
| Group BY Ability BY TIME | .09 | 2 | .04 | 2.76 | .071 |

Table A6

Analysis of Variance for Accuracy of Self-Evaluation of Performance (ASEP) with Repeated Measures on Time Factor

******Analysis of Variance--design 1******

Tests of Between-Subjects Effects.

Tests of Significance for T1 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 2.71 | 79 | .03 | | |
| Group | .11 | 2 | .05 | 1.54 | .221 |
| Ability | .20 | 1 | .20 | 5.75 | .019 |
| Group BY Ability | .03 | 2 | .01 | .38 | .682 |

******Analysis of Variance--design 1******

Tests involving 'TIME' Within-Subject Effect.

Tests of Significance for T2 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 1.73 | 79 | .02 | | |
| TIME | .14 | 1 | .14 | 6.35 | .014 |
| Group BY TIME | .01 | 2 | .00 | .15 | .858 |
| Ability BY TIME | .09 | 1 | .09 | 3.97 | .050 |
| Group BY Ability BY TIME | .00 | 2 | .00 | .11 | .893 |

Table A7

Analysis of Variance for the MPSPQ with Repeated Measures on Time Factor

******Analysis of Variance -- design 1******

Tests of Between-Subjects Effects.

Tests of Significance for T1 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 21126.30 | 91 | 232.16 | | |
| Group | 45.11 | 2 | 22.55 | .10 | .908 |
| Ability | 2467.90 | 1 | 2467.90 | 10.63 | .002 |
| Group BY Ability | 138.79 | 2 | 69.40 | .30 | .742 |

******Analysis of Variance -- design 1******

Tests involving 'TIME' Within-Subject Effect.

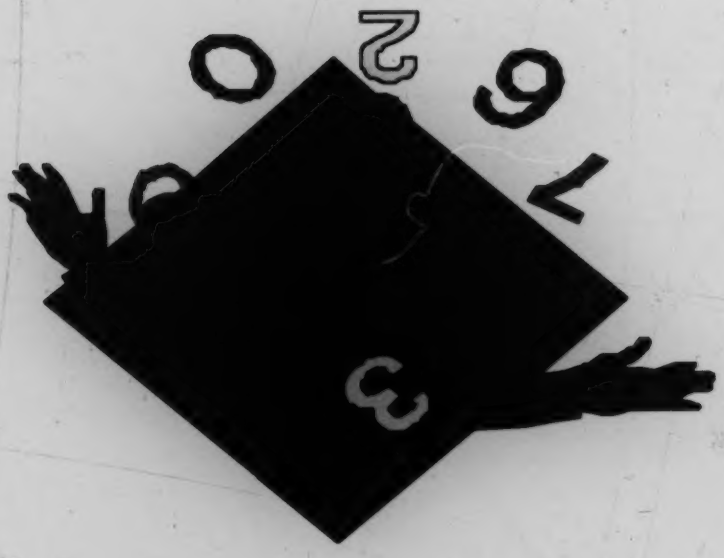
Tests of Significance for T2 using UNIQUE sums of squares

| Source of Variation | SS | DF | MS | F | Sig of F |
|---|---|---|---|---|---|
| WITHIN+RESIDUAL | 4690.47 | 91 | 51.54 | | |
| TIME | 2.40 | 1 | 2.40 | .05 | .830 |
| Group BY TIME | 49.98 | 2 | 24.99 | .48 | .617 |
| Ability BY TIME | 2.28 | 1 | 2.28 | .04 | .834 |
| Group BY Ability BY TIME | 156.29 | 2 | 78.14 | 1.52 | .225 |

# Appendix B

Mathematics Problem-Solving Process Questionnaire (MPSPQ)

# Mathematics Problem-Solving Process Questionnaire



**Name :** _____

**School :** _____

**Grade :** _____

**Gender:** _____

**Date:** _____

## To the teacher:

**Purpose:** This Math Problem-Solving Process Questionnaire is designed to examine students' cognitive (thinking) and metacognitive (what they know about their ability to think) processes, and their beliefs toward math word problem solving.

**Directions:**

1. This questionnaire consists of 24 questions about how students feel about math word problem solving.
2. Please be sure that each student has a copy of the questionnaire, a pencil, and an eraser.
3. It takes approximately 15 minutes to administer.
4. There is no right or wrong answer. For each question the student will have 5 choices. He or she should read each question and then choose only one answer according to the following key: (for those students who may have reading difficulties you may read the question and alternatives for them)

   a. describes me very well
   b. describes me well
   c. describes me somewhat
   d. does not really describe me
   e. does not describe me at all.

Give the following example to students to ensure understanding:

**Example:**

**When I fail to solve a problem, I don't try to figure out why it didn't work.**

If this does not really describe the student, then he or she would put a circle around **d**.

**a b c (d) e**

If this describes the student somewhat, he or she would put a circle around **c**.

**a b (c) d e**

**To the student:**

This Math Problem Solving Process Questionnaire is designed to gather information about your feelings regarding math problem-solving, and your use of strategies when solving problems in math.

1. This questionnaire consists of 24 questions about how you feel about problem solving in math.
2. Be sure that you have a copy of the questionnaire, an eraser and a pencil.
3. It takes approximately 15 minutes to complete.
4. There is no right or wrong answer. For each question you will have 5 choices. You are to read each question and then choose only one answer according to the following key:

   a. describes me very well
   b. describes me well
   c. describes me somewhat
   d. does not really describe me
   e. does not describe me at all.

   **Example**

   **When I fail to solve a problem, I don't try to figure out why it didn't work.**

   If this does not really describe you then you would put a circle around **d**. **a b c (d) e**

   If this describes you somewhat you would put a circle around **c**. **a b (c) d e**

5. Please erase completely if you change your mind, and circle the new answer.
6. Read carefully before you answer the question.

Name_____ Date_____

e. does not describe me at all
d. does not really describe me
c. describes me somewhat
b. describes me well
a. describes me very well

| | | | | | |
|---|---|---|---|---|---|
| 1. I underline the key words in math word problems. | a | b | c | d | e |
| 2. While solving a math problem, I use at least one strategy that helps me to find a solution. | a | b | c | d | e |
| 3. I have a step-by-step way to compare ideas and to make a plan when solving math problems. | a | b | c | d | e |
| 4. When I choose a way to answer a problem, I do not take time to think about whether all the possible answers could be right. | a | b | c | d | e |
| 5. When I read a question, I often skip over charts, graphs, and tables because other information is more important. | a | b | c | d | e |
| 6. I make quick decisions and later wish I hadn't. | a | b | c | d | e |

e. does not describe me at all
d. does not really describe me
c. describes me somewhat
b. describes me well
a. describes me very well

| | | | | | |
|---|---|---|---|---|---|
| 7. While solving math problems, I tend to skip difficult problems. | a | b | c | d | e |
| 8. I try to check the solution before turning to the next math problem. | a | b | c | d | e |
| 9. When I know that there is a problem, the first thing I do is to try and find out exactly what the problem is. | a | b | c | d | e |
| 10. When I try to solve problems, I don't usually think about using strategies. | a | b | c | d | e |
| 11. I think that problems can be explored and solved best with an organized method. | a | b | c | d | e |
| 12. After I have solved a problem, I do not think about what went right or what went wrong. | a | b | c | d | e |
| 13. Sometimes when I work on a math problem I feel like I'm not getting down to the real problem. | a | b | c | d | e |

**e**. does not describe me at all
**d**. does not really describe me
**c**. describes me somewhat
**b**. describes me well
**a**. describes me very well

14. I find that drawing diagrams helps me to solve problems. **a b c d e**

15. While solving math problems, comparing and contrasting the information is a waste of time. **a b c d e**

16. When I have a problem, I always look at how I feel about solving it, to get a better idea about the problem. **a b c d e**

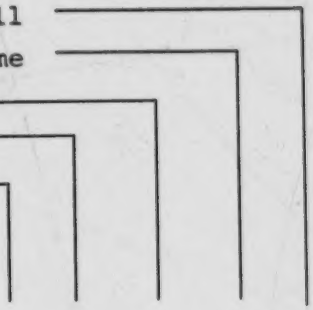
17. I always try to understand the important points in math word problems. **a b c d e**

18. I try out another strategy for problem solving if my first choice does not work. **a b c d e**

19. Before I begin to solve math word problems, I always write a brief outline. **a b c d e**

20. When I fail to solve a problem, I try to figure out why it didn't work. **a b c d e**

e. does not describe me at all
d. does not really describe me
c. describes me somewhat
b. describes me well
a. describes me very well



21. I have trouble understanding just what a math problem is asking. a b c d e

22. When I have difficult math problems, I don't take time to think of strategies to solve them. a b c d e

23. I have trouble doing basic calculations, such as addition subtraction, multiplication, and division. a b c d e

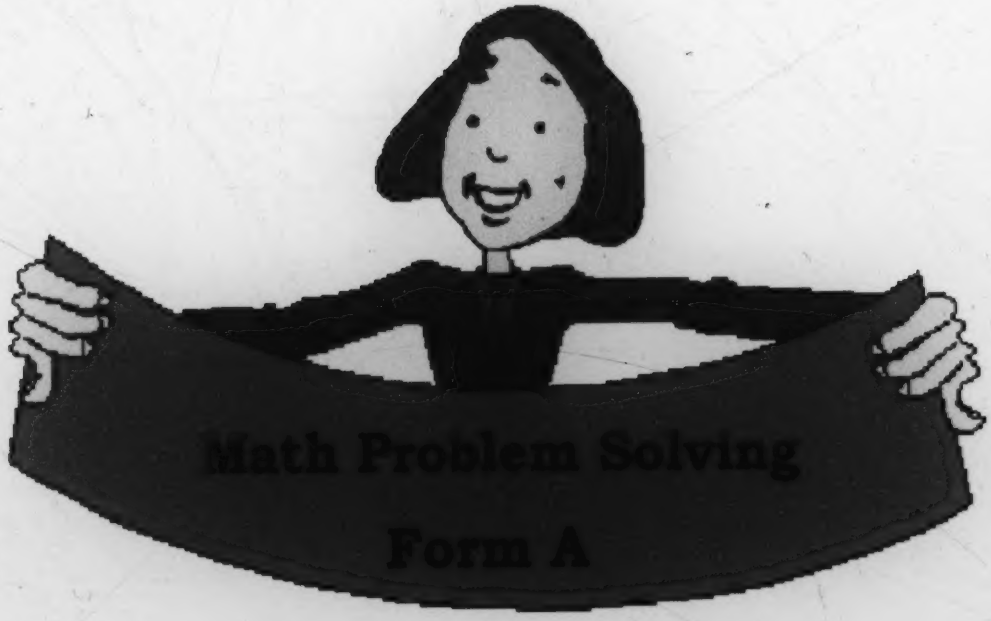
24. After reading a math question, I always stop to check whether I remember the question. a b c d e

Thank You!

# Appendix C

Mathematics Problem-Solving Test (MPST)

(Forms A & B)




Name: _____

School: _____

Grade: _____

Gender: _____

Date: _____

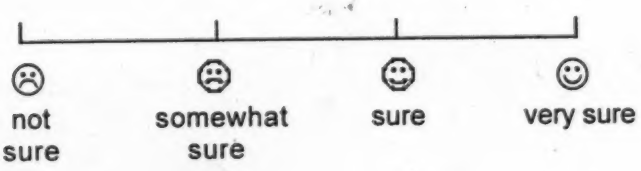
## Directions:

- This booklet contains three mathematical problems.
- Each of the three problems has several parts.
- These questions are to find out how well you can solve mathematical problems.
- **It is important to show how you work something out, so please explain your answer in the spaces on the question paper.** You can use the blank pages if you need more space.

## Question 1

For her sixth birthday party, Kelly's mother baked her favorite blueberry cheese cake. At the party, Frank ate 1/3 of the cake, Ann ate 1/6 of it, and Harry ate 1/4 of it. If Kelly ate 6 pieces of cake, how many pieces of cake did each person eat, and how many pieces of cake did they eat altogether?

A) How sure are you that you will get the answer correct?



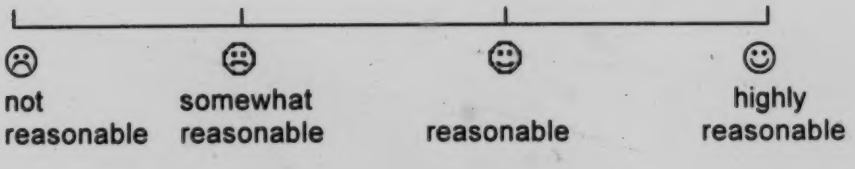

B) What was the question?

C) Write down all the information you think you need to solve the problem.

D) What strategy will help you solve the problem?

E) Now, solve the problem showing all of your work .

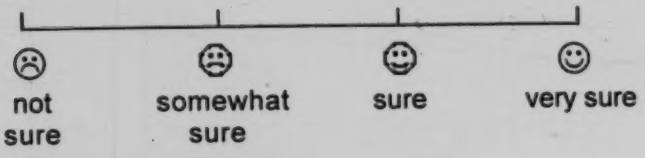
G) How reasonable do you think your solution was?



## Question 2

Jerry, an Edmonton bus tour guide, was getting a tour group in from London, England. He wanted to know how many total seats there were on the bus. He counted 31 people and noticed that 1 out of every 4 seats was empty. How many empty seats were there on the bus? (Do not include the bus driver.)

A) How sure are you that you will get the answer correct?



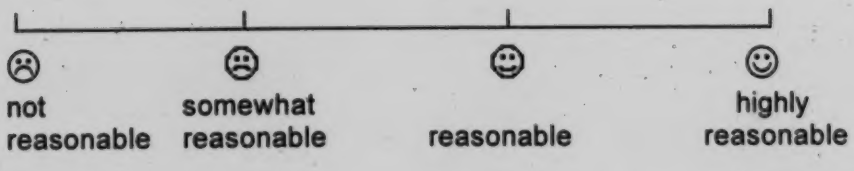

B) What was the question?

C) Write down all the information you think you need to solve the problem.

D) What strategy will help you solve the problem?

E) Now, solve the problem showing all of your work .

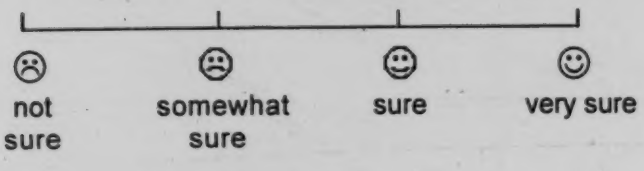
G) How reasonable do you think your solution was?



## Question 3

Joyce, Tanya, Judy, and Darleen are quadruplets, and the only way their mother can tell them apart is by the color of their skirts. Tanya always wears red. Joyce and Darleen do not have purple skirts. Neither Tanya nor Joyce likes beige. What color skirt does each girl prefer: red, purple, beige, blue?

A) How sure are you that you will get the answer correct?



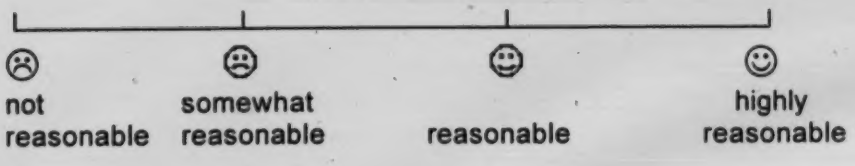

B) What was the question?

C) Write down all the information you think you need to solve the problem.

D) What strategy will help you solve the problem?

E) Now, solve the problem showing all of your work .

G) How reasonable do you think your solution was?



Name: _____

School: _____

Grade: _____

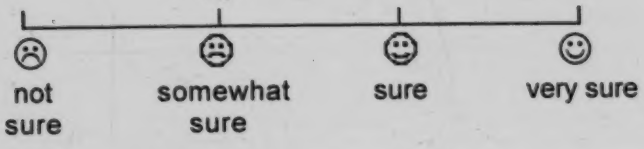
Gender: _____

Date: _____

## Question 1

For his twelfth birthday party, Rob's mother made a pepperoni pizza. At the party, Jane ate 1/3 of the pizza, Nick ate 1/6 of it, and Maureen ate 1/4 of it. If Rob ate 8 pieces of pizza, how many pieces of pizza did each person eat, and how many pieces of pizza did they eat altogether?

How sure are you that you will get the answer correct?



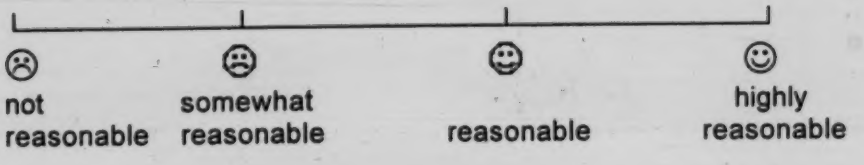

B) What was the question?

C) Write down all the information you think you need to solve the prelim.

D) What strategy will help you solve the problem?

E) Now, solve the problem showing all of your work .

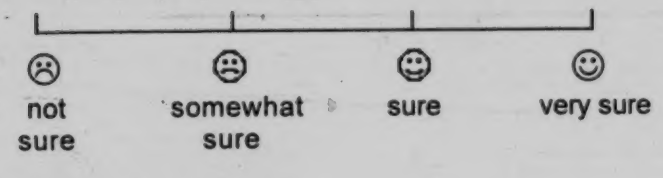
G) How reasonable do you think your solution was?



**Question 2**

On Saturday, Jessica had a part-time job as an apple inspector at the Sunshine Orchard. She wanted to know how many good apples there were at the orchard. She found 39 good apples and noticed that 1 out of every 4 apples had worm holes. How many apples had worm holes?

A) How sure are you that you will get the answer correct?



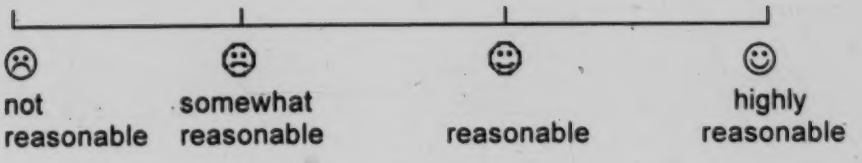

B) What was the question?

C) Write down all the information you think you need to solve the problem.

D) What strategy will help you solve the problem?

E) Now, solve the problem showing all of your work .

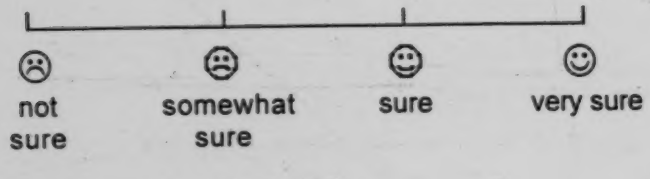
G) How reasonable do you think your solution was?



## Question 3

Scott, Kurt, Eric, and Andrew are quadruplets, and the only way their mother can tell them apart is by the color of their shirts. Kurt always wears orange. Scott and Andrew do not have green shirts. Neither Kurt nor Scott likes brown. What color shirt does each boy prefer: orange, green, brown, yellow?

A) How sure are you that you will get the answer correct?



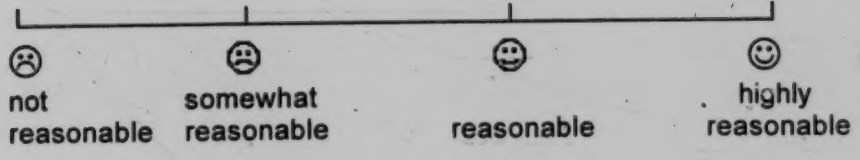

B) What was the question?

C) Write down all the information you think you need to solve the problem.

D) What strategy will help you solve the problem?

E) Now, solve the problem showing all of your work .

G) How reasonable do you think your solution was?



# Appendix D

## Attitude Survey

Name _____ Date _____

e. does not describe me at all
d. does not really describe me
c. describes me somewhat
b. describes me well
a. describes me very well

| | | | | | |
|---|---|---|---|---|---|
| 1. I like math. | a | b | c | d | e |
| 2. I like math problem-solving. | a | b | c | d | e |
| 3. I am interested in using the computer. | a | b | c | d | e |
| 4. I didn't like math problem-solving before I tried Doehee's math lessons. | a | b | c | d | e |
| 5. It was fun to try Doehee's math lessons. | a | b | c | d | e |
| 6. I felt that the computer was my private tutor. | a | b | c | d | e |
| 7. Taking Doehee's math lessons made it possible for me to work at my own pace. | a | b | c | d | e |
| 8. Doehee's math lessons made me more interested in math problem-solving. | a | b | c | d | e |

e. does not describe me at all
d. does not really describe me
c. describes me somewhat
b. describes me well
a. describes me very well

9. I like Doehee's math lessons better than classroom instruction. a b c d e

10. Doehee's math lessons helped me understand math problem-solving better. a b c d e

11. Doehee's math lessons made me like math problem-solving better. a b c d e

12. I came to like math problem-solving after I tried Doehee's math lessons. a b c d e

* Please make any comment in the space below which can help explain your feeling about Doehee's math lessons. You can use the blank pages if you need more space.